Modeling Multistability and Hysteresis in Devices

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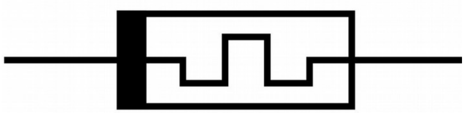
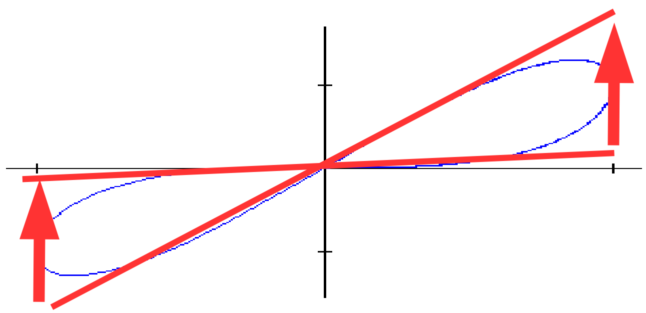
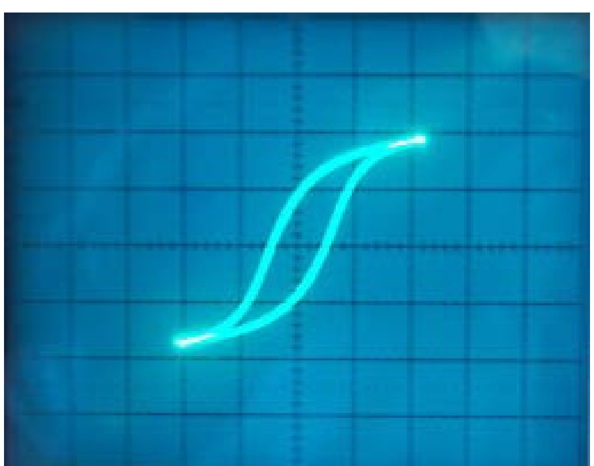
Devices with Hysteresis



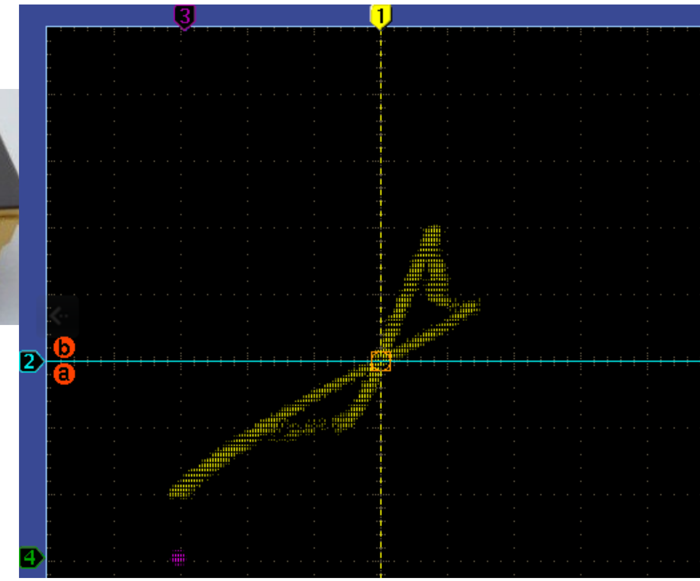
memristor

(RRAM, CBRAM, PCM…)

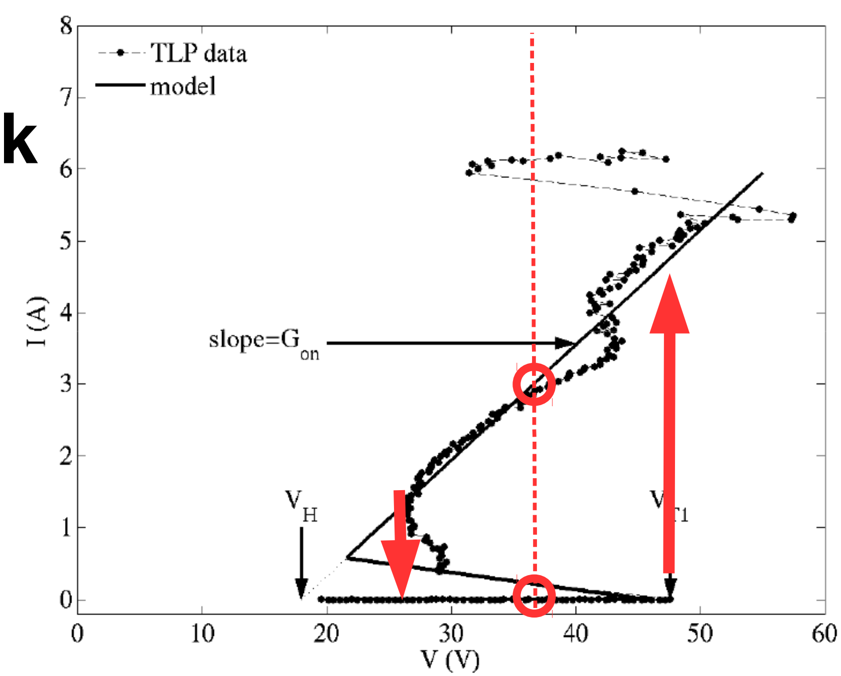




magnetic core



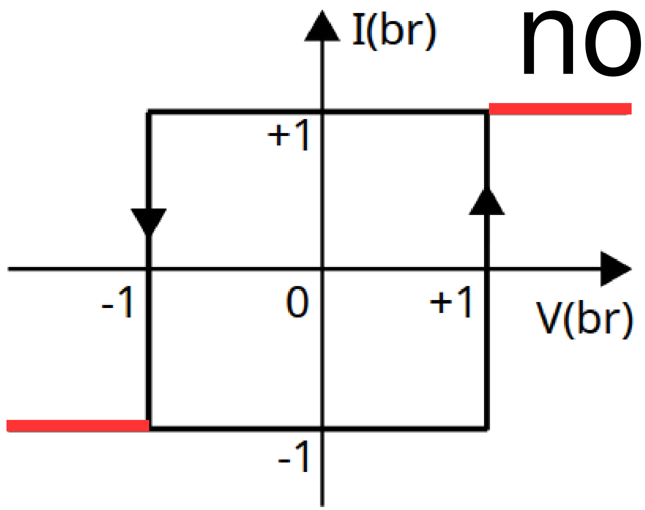
ESD snapback



Ida/McAndrew. "A Physically-based Behavioral Snapback Model." IEEE EOS/ESD Symposium, 2012.

How ^ to model hysteresis?

not



"memory state"
"hidden state"

```
real i;
analog begin
    if V(br) < -1
        i = -1;
    if V(br) > +1
        i = +1;
    I(br) <+ i;
end
```

not an analog compact model

memristor
(RRAM, CBRAM, PCM...)

- Linear/nonlinear ion drift models
  Biolek (2009), Jogelkar(2009), Prodromakis (2011), *etc.*
- UMich RRAM model (2011)
- TEAM model (2012)
- Simmons tunneling barrier model (2013)
- Yakopcic model (2013)
- Stanford/ASU RRAM model (2014)
- Knowm "probabilistic" model (2015)

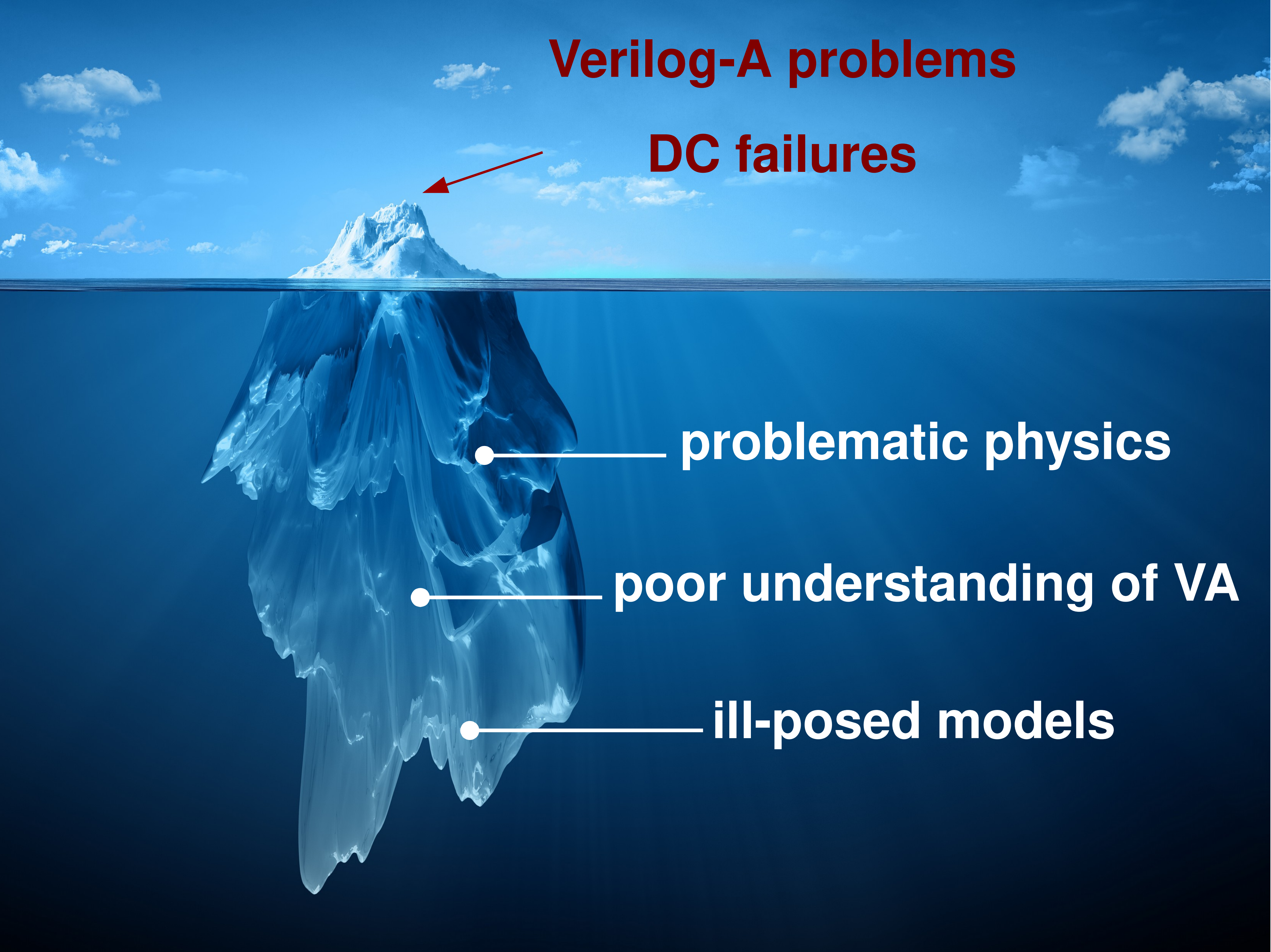
```
$bound_step(tstep);
c_time = $abstime;
dt = c_time - p_time;
x = x_last + dt * exp(…);
```

```
@(initial_step) begin
    x = x_init;
end
```

Boolean variable "hybrid model"

```
int isON = 0;
if (abs(V(…)) > V_snap)
    isON = 1;
if (isON) {
    ...
} else {
    ...
}
```

only for TRAN
none works for DC, AC, PSS


Verilog-A problems
DC failures
problematic physics
poor understanding of VA
ill-posed models

Good Compact Models

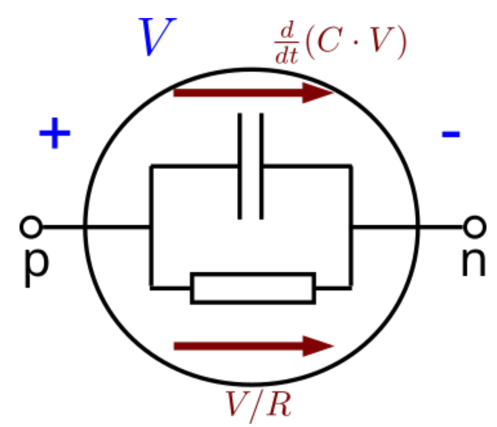
- "simulation-ready"
  - run in all analyses (DC, AC, TRAN, sensitivity, shooting, HB, …)
  - run in all simulators

consistently

~~analysis-specific code~~

- a simple (trivial) example

```
...
I(p, n) <+ V(p, n)/R;
I(p, n) <+ ddt(C * V(p, n));
...
```



  - differential equation format

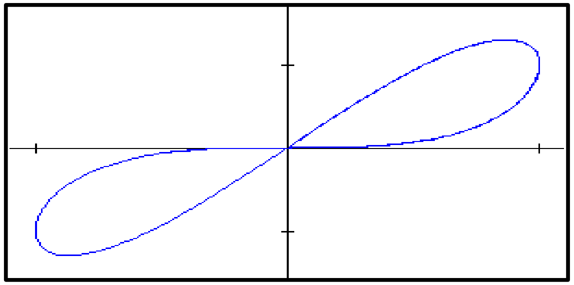
$$\text{ipn} = \frac{d}{dt} q(\text{vpn}) + f(\text{vpn})$$

    "charges" and "currents", continuous and smooth

  - no $abstime, idt(), @initial_step, @cross, $bound_step, *etc.*
  - <u>well-posed</u>
    - a solution exists
    - the solution is unique
    - the solution's behavior changes continuously with the initial conditions.

https://en.wikipedia.org/wiki/Well-posed_problem

How to model hysteresis?

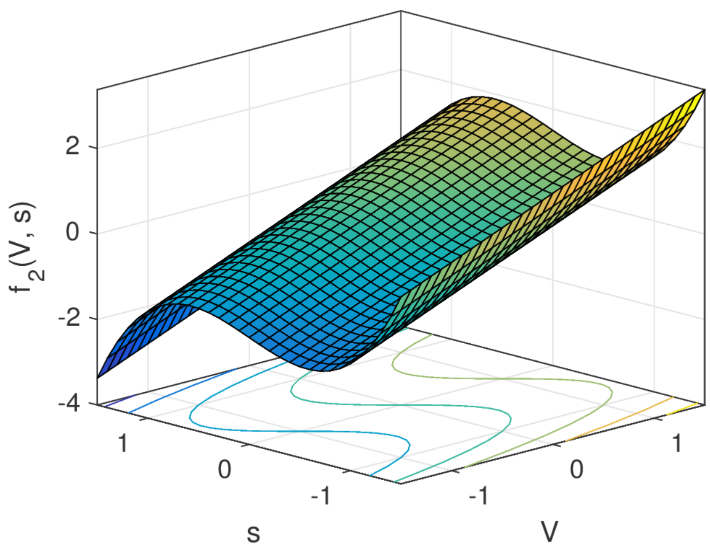


- hysteresis ≠ discontinuity or if-else
- hysteresis ≠ use $abstime
  - or @(initial_step), $bound_step, *etc.*
- hysteresis ≠ hybrid models
- hysteresis/multistability ≠ "flat" regions w zero derivatives

⇕

- model hysteresis using internal state variable
  - proper design of dynamics
- write internal unknown in Verilog-A
  - use potentials/flows
- set bounds for internal unknown with equations
  - physical distance
  - clipping functions
- smoothness, continuity, finite precision issues, …

How to Model Hysteresis Properly

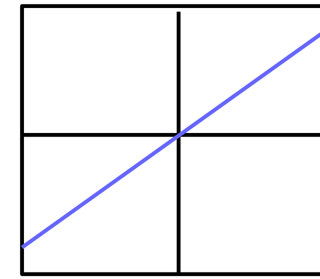


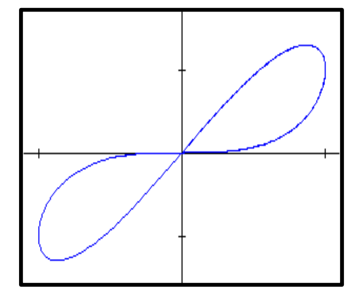
$$\mathbf{ipn} = f(\mathbf{vpn})$$

$$\updownarrow$$

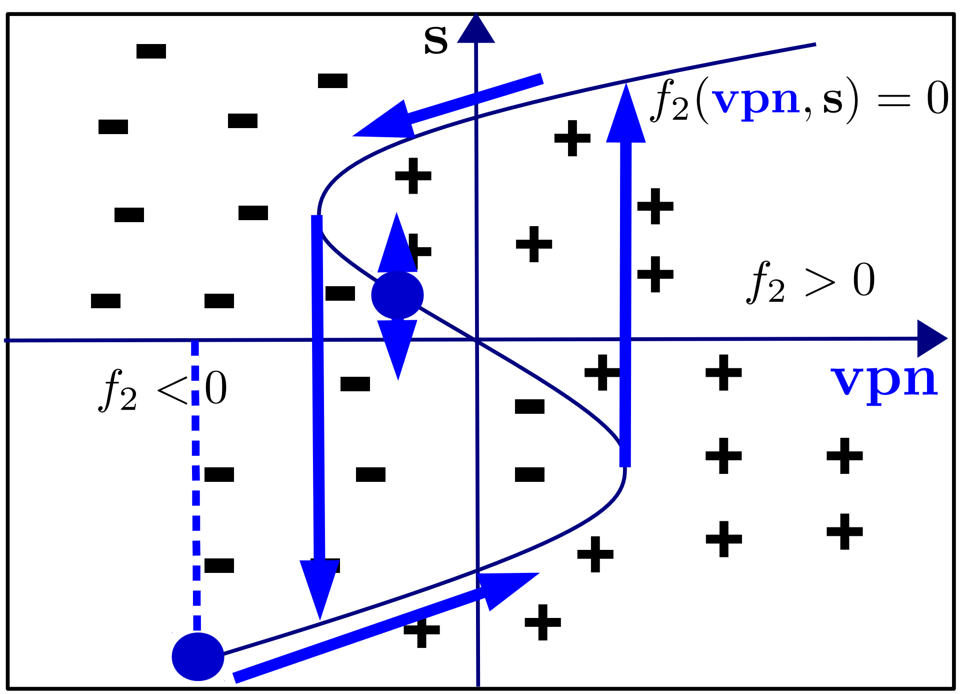
$$\mathbf{ipn} = f_1(\mathbf{vpn},\ \mathbf{s})$$

$$\frac{d}{dt}\mathbf{s} = f_2(\mathbf{vpn},\ \mathbf{s})$$





internal state variable
"memory"



Example:

$$f_1(\mathbf{vpn},\ \mathbf{s}) = \frac{\mathbf{vpn}}{R} \cdot (1 + \tanh(\mathbf{s}))$$

$$f_2(\mathbf{vpn},\ \mathbf{s}) = \mathbf{vpn} - \mathbf{s}^3 + \mathbf{s}$$

multistability → abrupt change in DC sols

negative-sloped fold → hysteresis

How to Model Hysteresis Properly

Template:

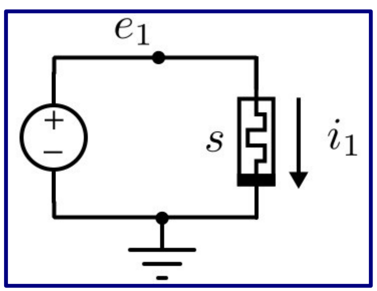
$$\mathbf{ipn} = f_1\left(\mathbf{vpn},\ \mathbf{s}\right)$$

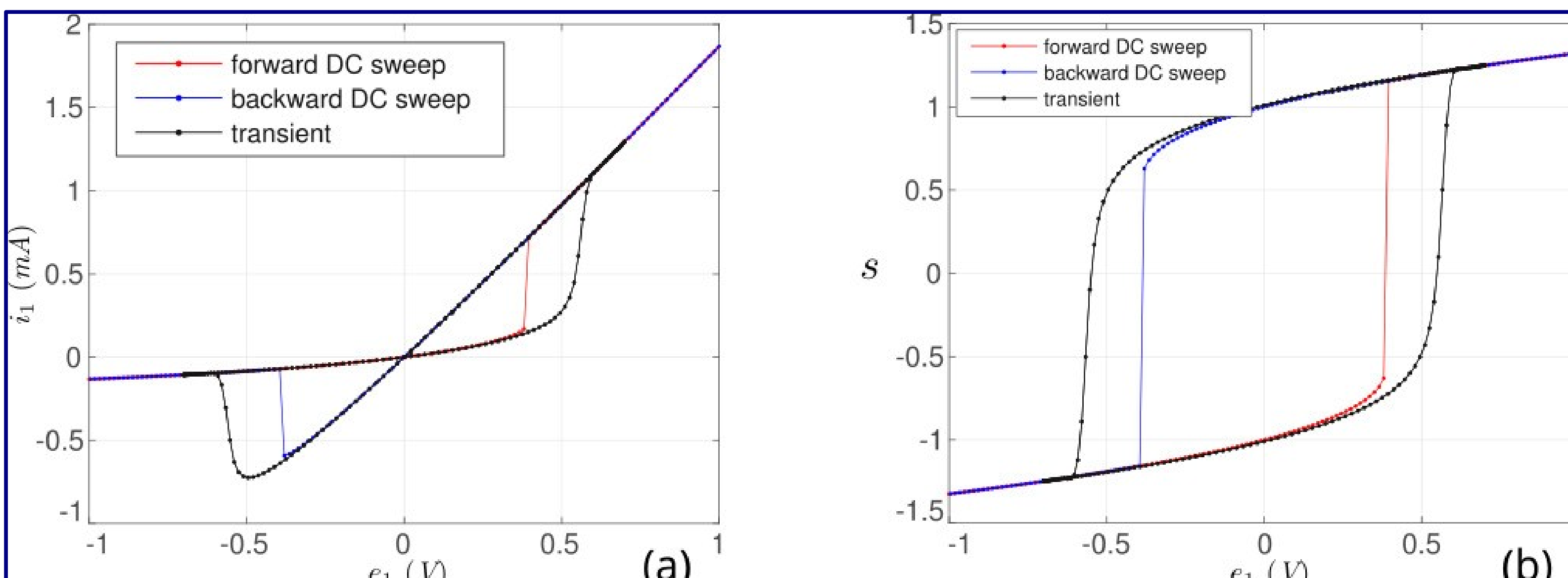
$$\frac{d}{dt}\mathbf{s} = f_2\left(\mathbf{vpn},\ \mathbf{s}\right)$$

MAPP (our internal simulator)**:**

$$\mathbf{ipn} = \frac{d}{dt}\underbrace{q_e\left(\mathbf{vpn},\ \mathbf{s}\right)}_{\mathbf{0}} + \underbrace{f_e\left(\mathbf{vpn},\ \mathbf{s}\right)}_{f_1}$$

$$0 = \frac{d}{dt}\underbrace{q_i\left(\mathbf{vpn},\ \mathbf{s}\right)}_{-\mathbf{s}} + \underbrace{f_i\left(\mathbf{vpn},\ \mathbf{s}\right)}_{f_2}$$

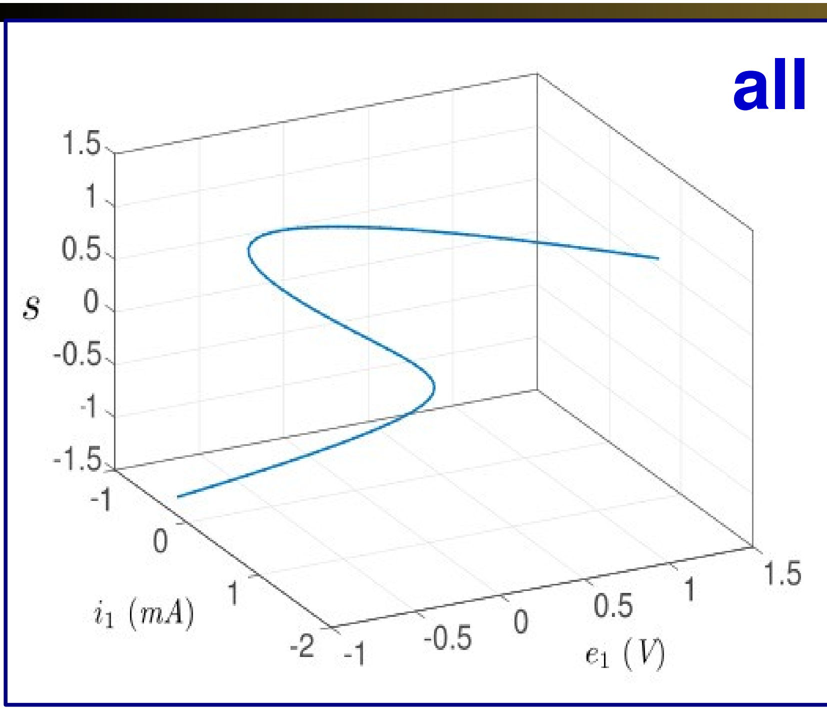




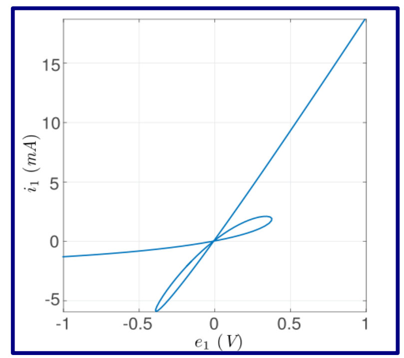
How to Model Hysteresis Properly

all DC sols from homotopy analysis

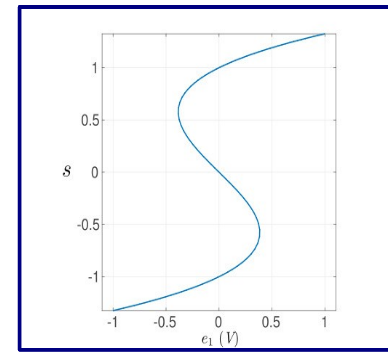
(like a curve tracer)

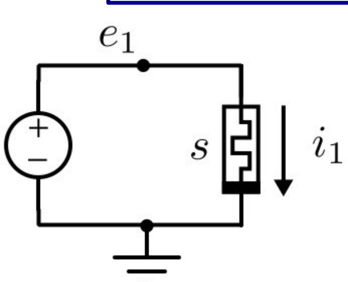


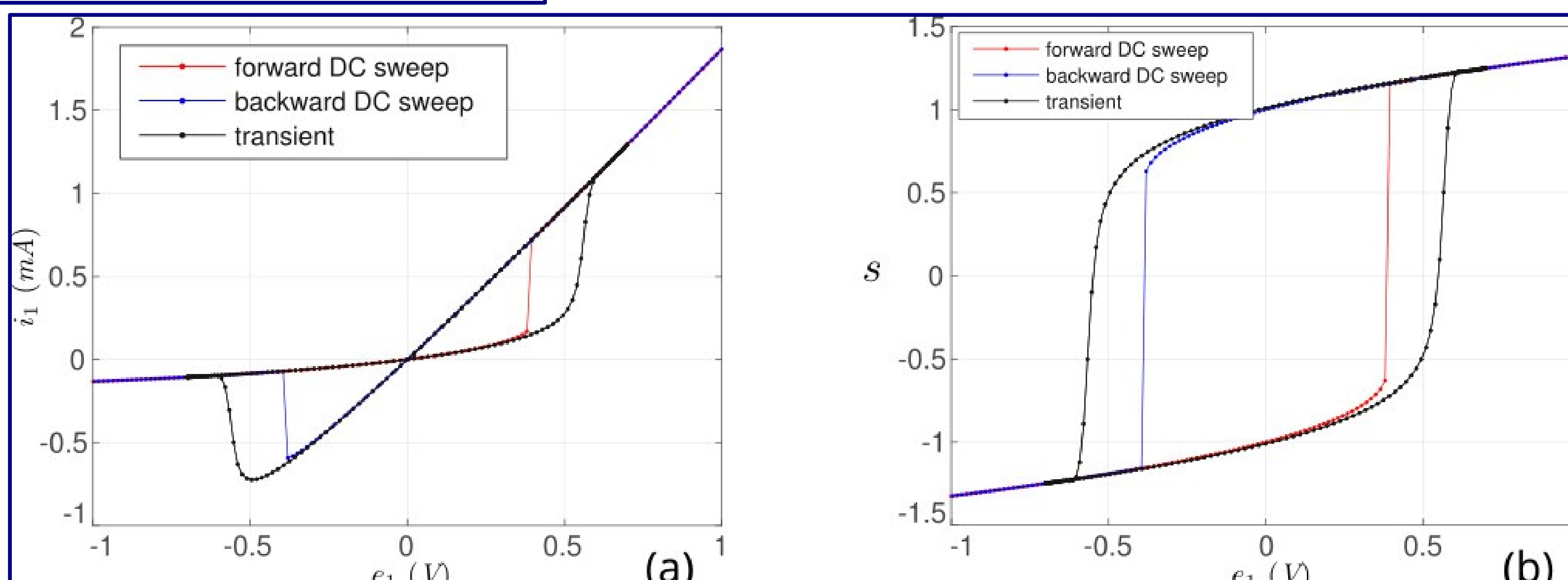
top



side







Internal Unknowns in Verilog-A

Template:

$$\mathbf{ipn} = f_1(\mathbf{vpn}, \mathrm{s})$$

$$\frac{d}{dt}\mathrm{s} = f_2(\mathbf{vpn}, \mathrm{s})$$

?

Example:

$$f_1(\mathbf{vpn}, \mathrm{s}) = \frac{\mathbf{vpn}}{R} \cdot (1 + \tanh(\mathrm{s}))$$

$$f_2(\mathbf{vpn}, \mathrm{s}) = \mathbf{vpn} - \mathrm{s}^3 + \mathrm{s}$$

DO NOT

- declare internal unknowns as "`real`" variables
- code time integration inside model
  - with `$abstime`, `@cross`, `@initial_step`, memory states
- use `idt()`
- use implicit contributions
  - unless you know what you are doing

Internal Unknowns in Verilog-A

$$\mathbf{ipn} = \frac{\mathbf{vpn}}{R} \cdot (1 + \tanh(\mathbf{s}))$$

$$\frac{d}{dt}(\tau \cdot \mathbf{s}) = \mathbf{vpn} - \mathbf{s}^3 + \mathbf{s}$$

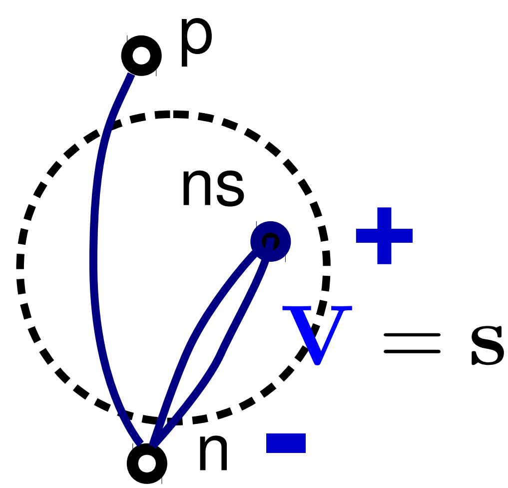
use a potential or flow

```
`include "disciplines.vams"
module hys(p, n);
    inout p, n;
    electrical p, n, ns;
    branch (ns, n) ns_br1;
    branch (ns, n) ns_br2;
    parameter real R = 1e3 from (0:inf);
    parameter real k = 1 from (0:inf);
    parameter real tau = 1e-5 from (0:inf);
    real s;

    analog begin
        s = V(ns, n);
        I(p, n)  <+ V(p, n)/R * (1+tanh(k*s));
        I(ns_br1) <+ V(p, n) - pow(s, 3) + s;
        I(ns_br2) <+ ddt(-tau*s);
    end
endmodule
```

internal node

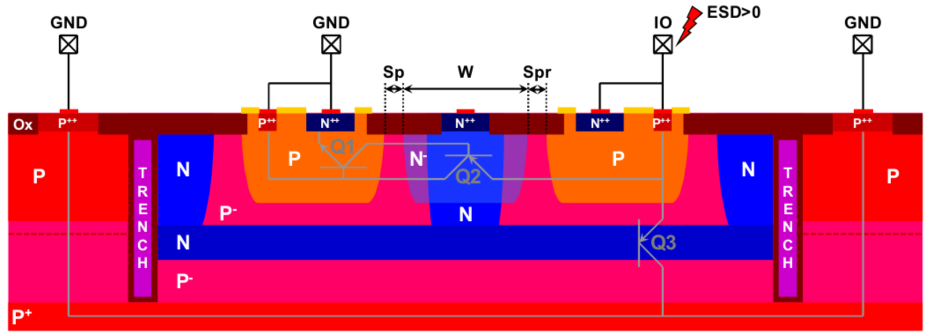
internal unknown

implicit differential equation

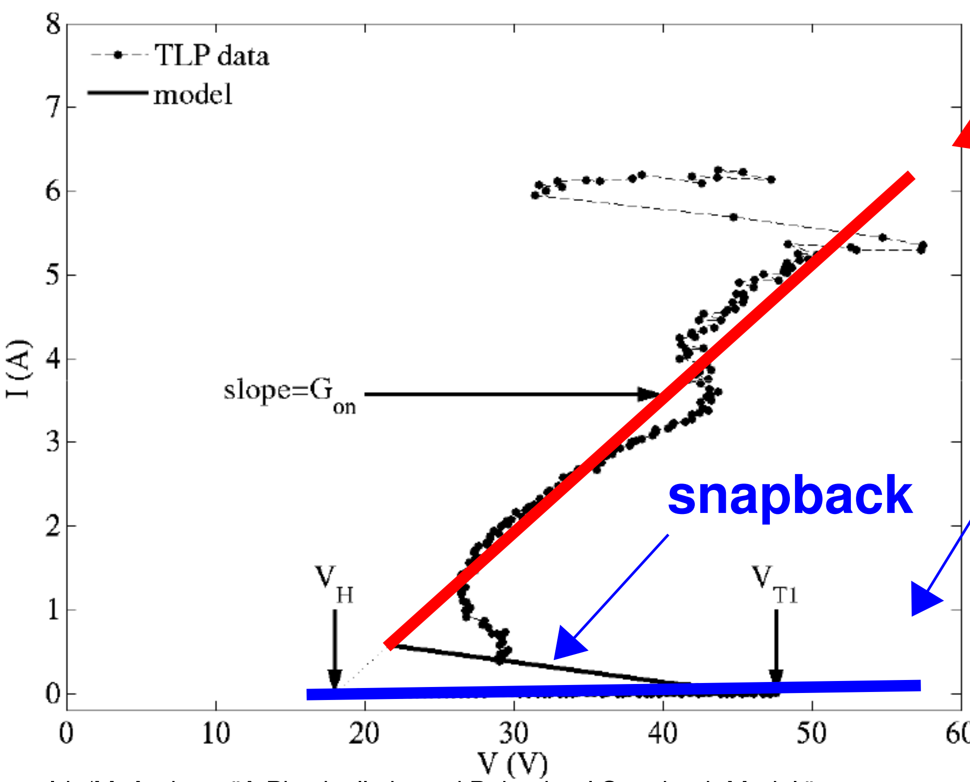


ESD Snapback Model

ESD protection device



Gendron, et al. "New High Voltage ESD Protection Devices based on Bipolar Transistors for Automotive Applications." IEEE EOS/ESD Symposium, 2011.



Ida/McAndrew. "A Physically-based Behavioral Snapback Model." IEEE EOS/ESD Symposium, 2012.

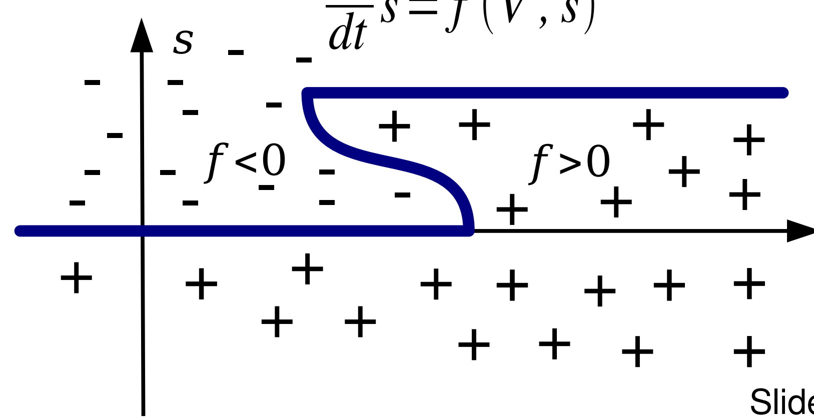
$$I_{on} = G_{on} \cdot (V - V_H)$$

$$I_{off} = I_S \cdot (1 - e^{-V/\phi_T}) \cdot \sqrt{1 + \frac{max(V, 0)}{V_D}}$$

$$I = s \cdot I_{on} + I_{off}$$

internal state: indicator of impact ionization

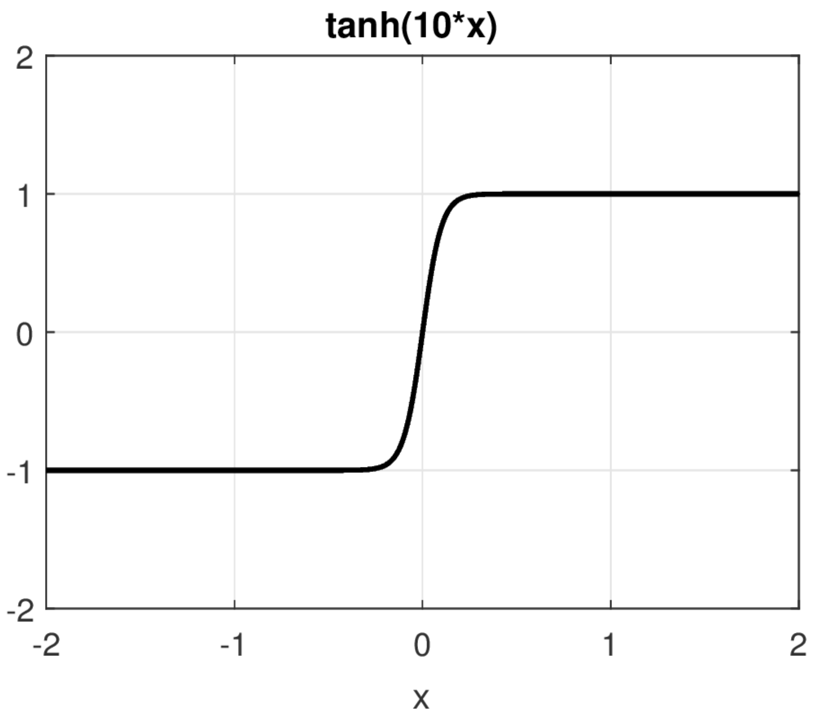
$$\frac{d}{dt} s = f(V, s)$$

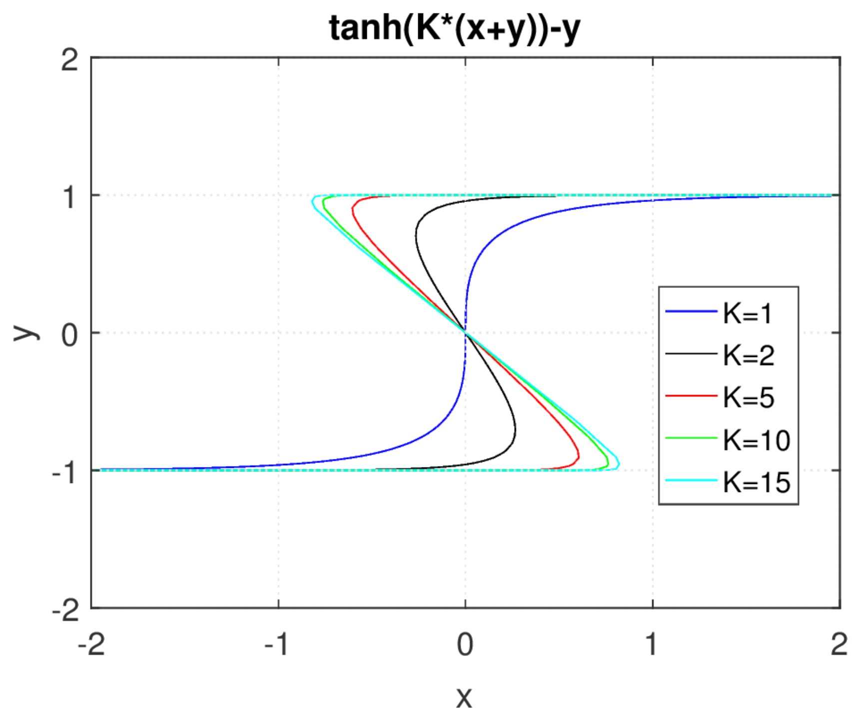


ESD Snapback Model

$$\frac{d}{dt}s = f\left(V, s\right)$$

many possible functions






```
...
Vstar = 2*(V(p, n)-0.5*VT1-0.5*VIH)/(VT1-VIH);
sstar = 2*(s-0.5);
I(ns, n) <+ tanh(K*(Vstar + sstar)) - sstar;
...
```

shift transition points

shift range to (0, 1)

ESD Snapback Model

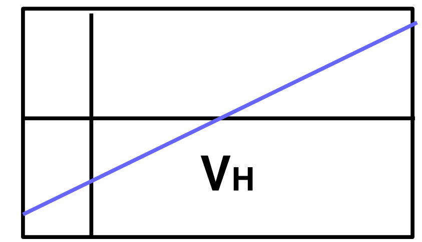
```
`include "disciplines.vams"

module ESDclamp(p, n);
    inout p, n;
    electrical p, n, ns;

    parameter real Gon = 0.1 from (0:inf);
    ...

    analog begin
        s = V(ns, n);
        Ion = smoothclip(Gon*(V(p, n)-VH), smoothing)
                    - smoothclip(-Gon*VH, smoothing);
        Ioff = Is * (1 - limexp(-V(p, n)/VT))
                     * sqrt(1 + max(V(p, n), 0)/VD);
        I(p, n) <+ Ioff + pow(s, Alpha) * Ion;
        I(p, n) <+ ddt(C * V(p, n));

        Vstar = 2*(V(p, n)-0.5*VT1-0.5*VIH)/(VT1-VIH);
        sstar = 2*(s-0.5);
        I(ns, n) <+ tanh(K*(Vstar + sstar)) - sstar;
        I(ns, n) <+ ddt(-tau*s);
    end
endmodule
```

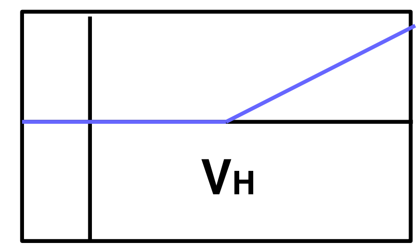
internal unknown as a voltage

Ion

Ioff

implicit differential equation



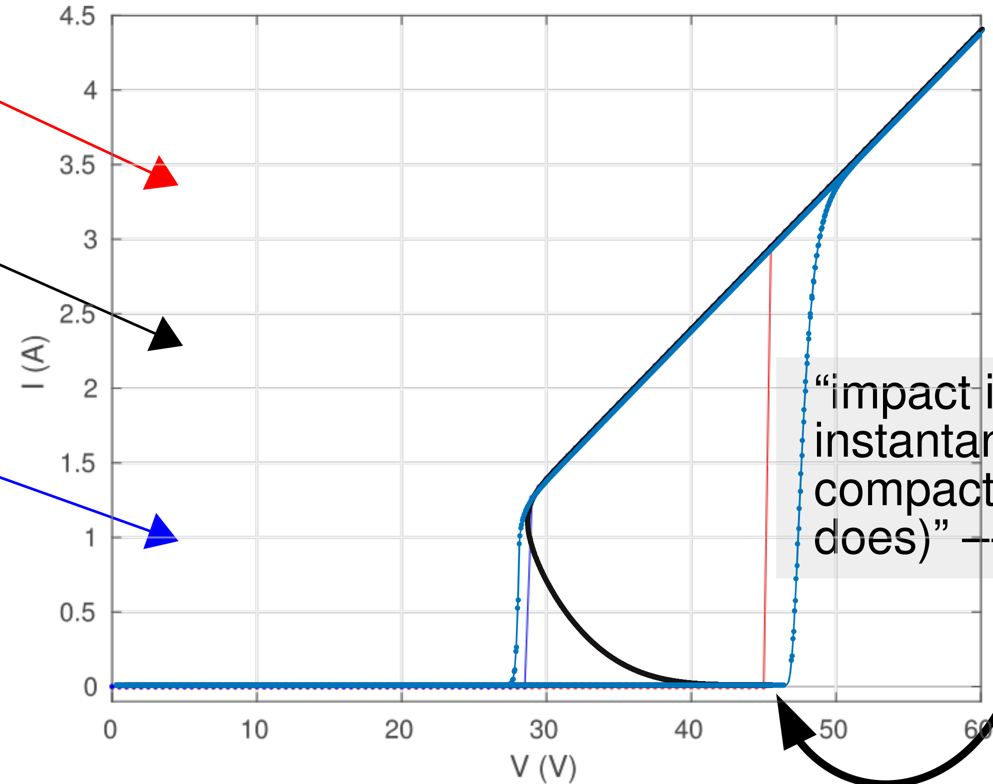


ESD Snapback Model

forward/backward DC sweeps

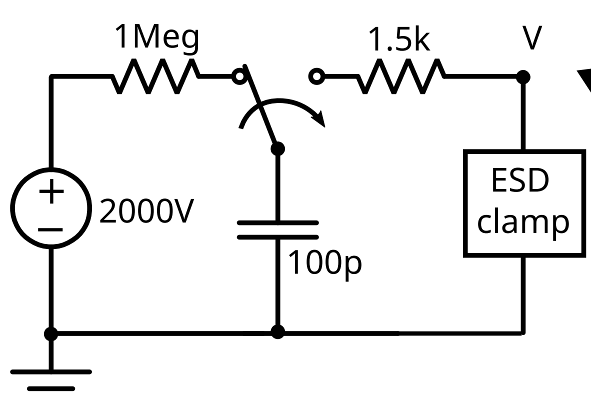
homotopy (all DC sols)

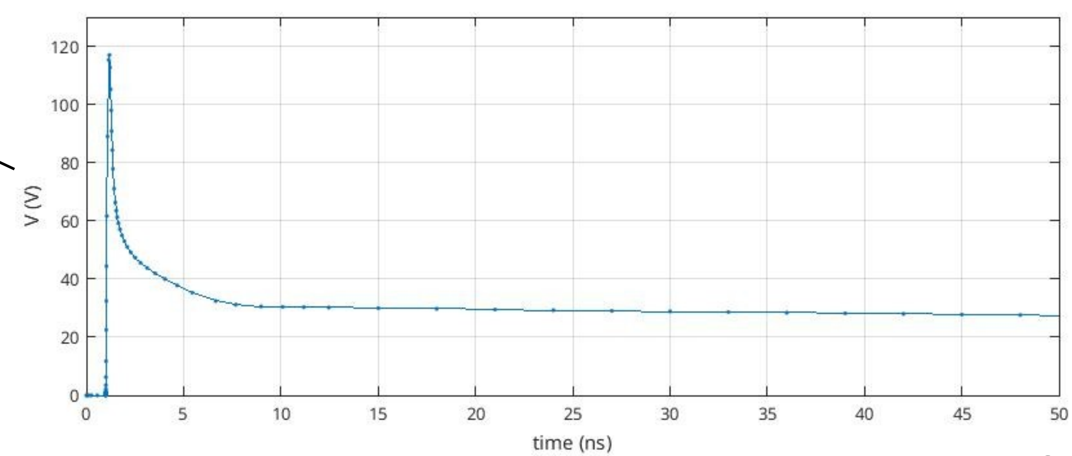
transient voltage sweeps



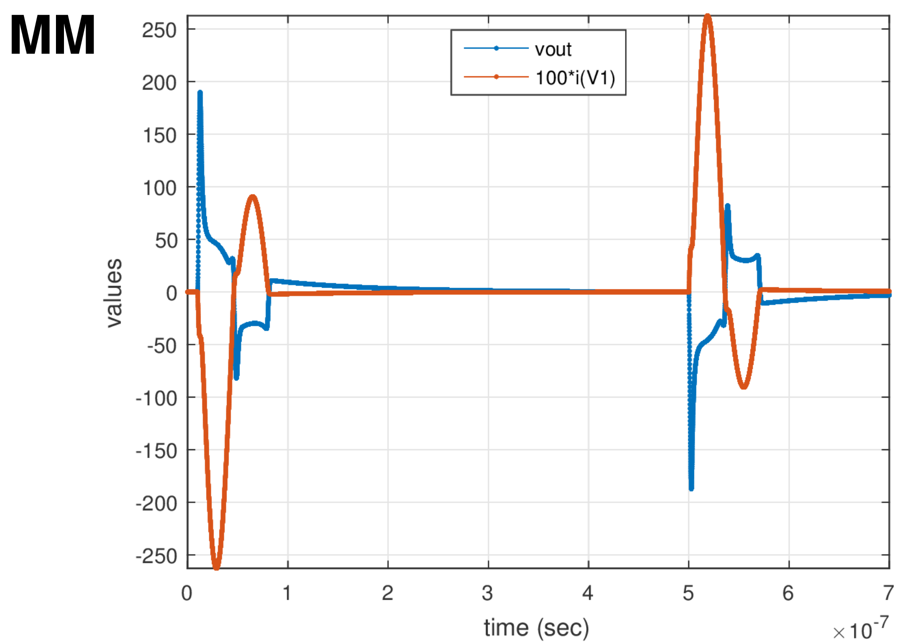
"impact ionization doesn't happen instantaneously (although all compact models assume that it does)" --- C.C. McAndrew

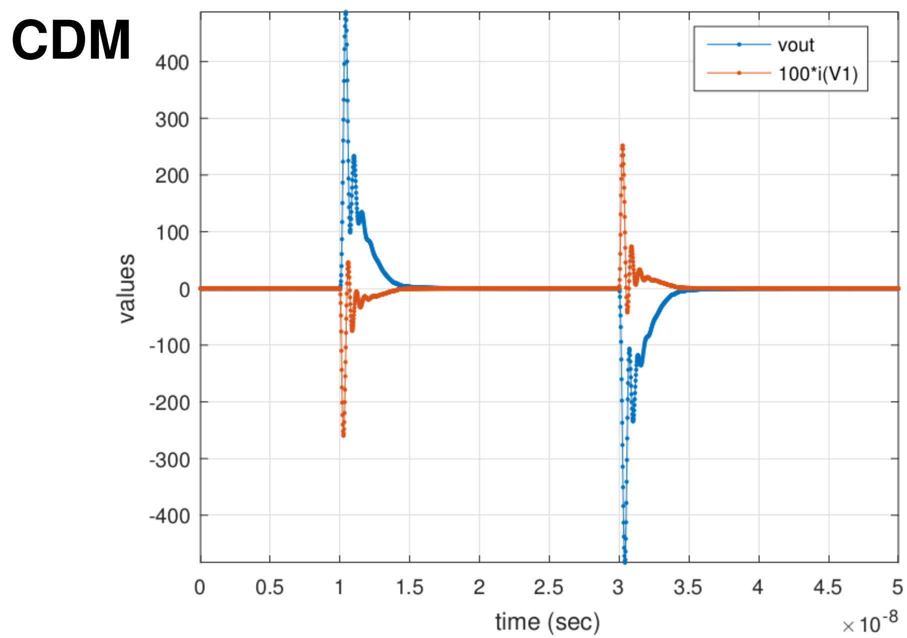
Human Body Mode (HBM) test

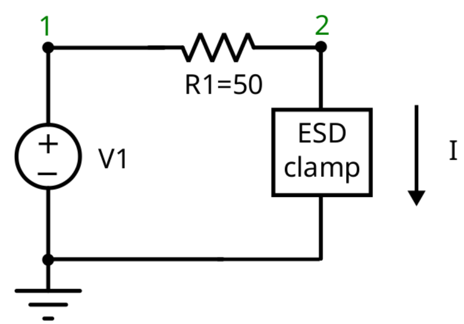




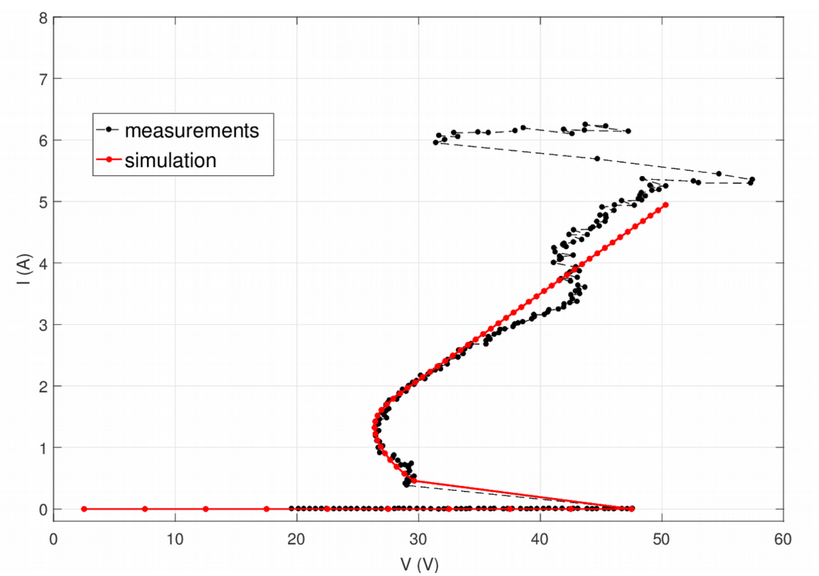
ESD Snapback Model

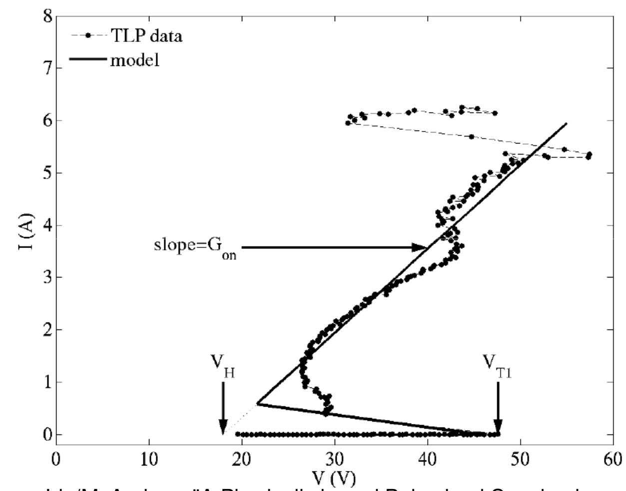






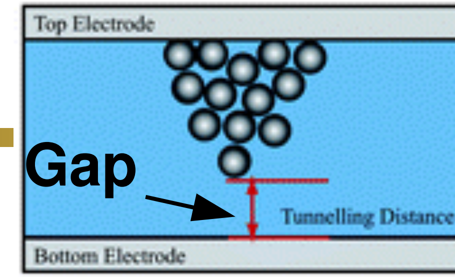
simple TLP equivalent circuit





Ida/McAndrew. "A Physically-based Behavioral Snapback Model." IEEE EOS/ESD Symposium, 2012.

RRAM Model



Template:

$$\mathbf{ipn} = f_1(\mathbf{vpn},\ \mathbf{s})$$

$$\frac{d}{dt}\mathbf{s} = f_2(\mathbf{vpn},\ \mathbf{s})$$

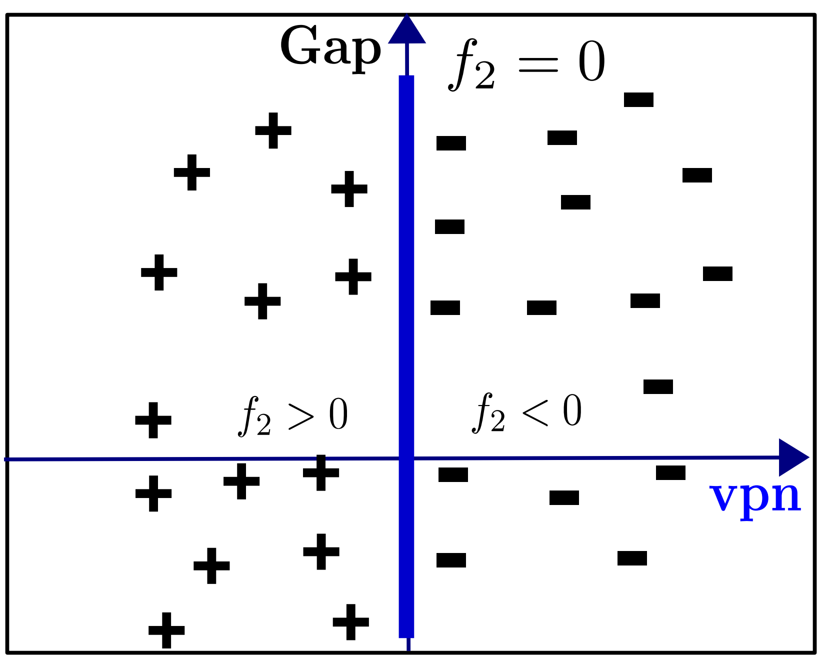
RRAM:

$$f_1(\mathbf{vpn},\ \mathbf{Gap}) = I_0 \cdot e^{-\mathbf{Gap}/g0} \cdot \sinh(\mathbf{vpn}/V_0)$$

$$f_2(\mathbf{vpn},\ \mathbf{Gap}) = -v_0 \cdot \exp(-\frac{E_a}{V_T}) \cdot \sinh(\frac{\mathbf{vpn} \cdot \gamma \cdot a_0}{t_{ox} \cdot V_T})$$

Jiang, Z., Wong, H. (2014). Stanford University Resistive-Switching Random Access Memory (RRAM) Verilog-A Model. nanoHUB.

$$\mathbf{minGap} \leq \mathbf{Gap} \leq \mathbf{maxGap}$$



~~if gap < minGap~~
~~gap = minGap;~~

hybrid model

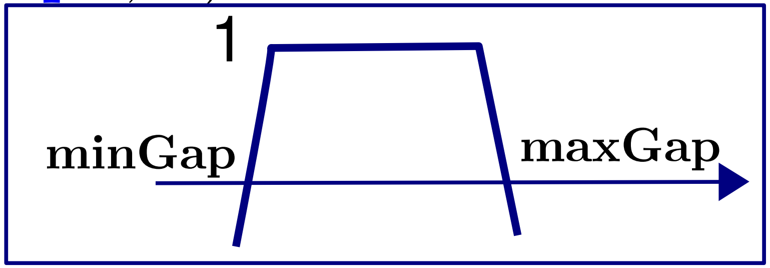
RRAM Model

Template: **RRAM:**

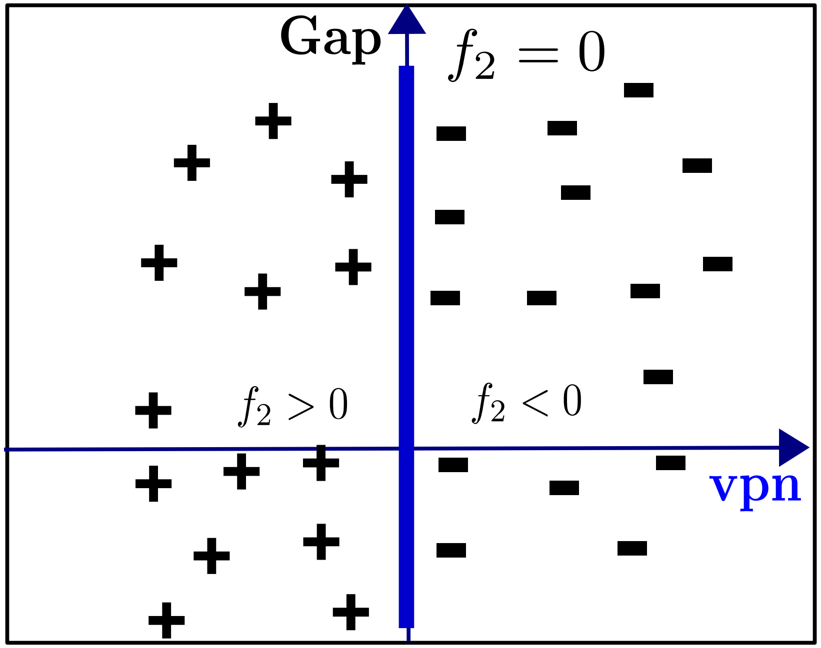
$$\mathbf{ipn} = f_1(\mathbf{vpn},\ \mathbf{s}) \qquad f_1(\mathbf{vpn},\ \mathbf{Gap}) = I_0 \cdot e^{-\mathbf{Gap}/g0} \cdot \sinh(\mathbf{vpn}/V_0)$$

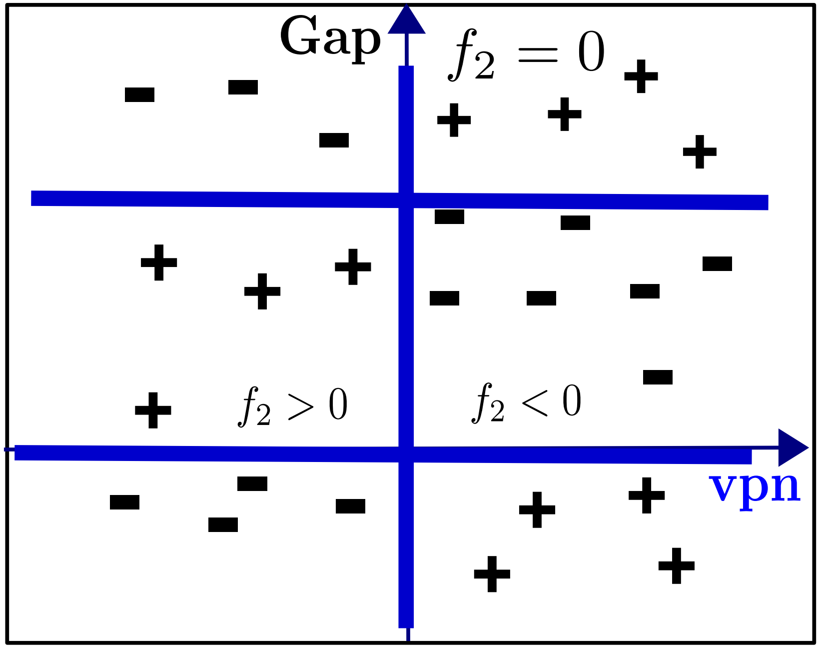
$$\frac{d}{dt}\mathbf{s} = f_2(\mathbf{vpn},\ \mathbf{s}) \qquad f_2(\mathbf{vpn},\ \mathbf{Gap}) = -v_0 \cdot \exp(-\frac{E_a}{V_T}) \cdot \sinh(\frac{\mathbf{vpn} \cdot \gamma \cdot a_0}{t_{ox} \cdot V_T})$$

$$\times F_{window}(\mathbf{Gap})$$

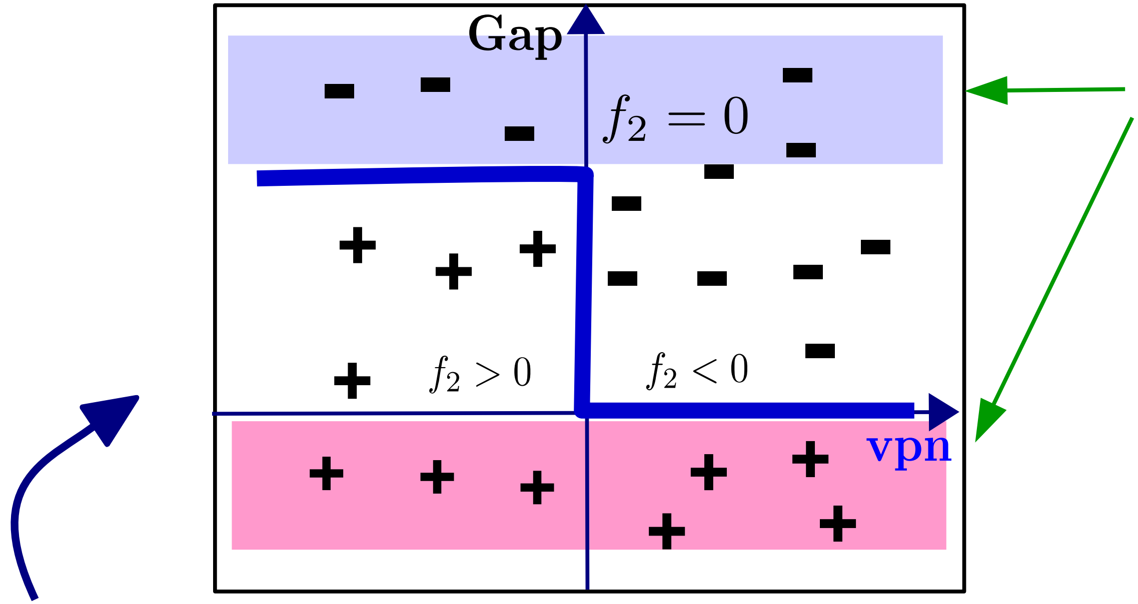


Biolek, Jogelkar, Prodromakis, UMich, TEAM/VTEAM, Yakopcic, etc.



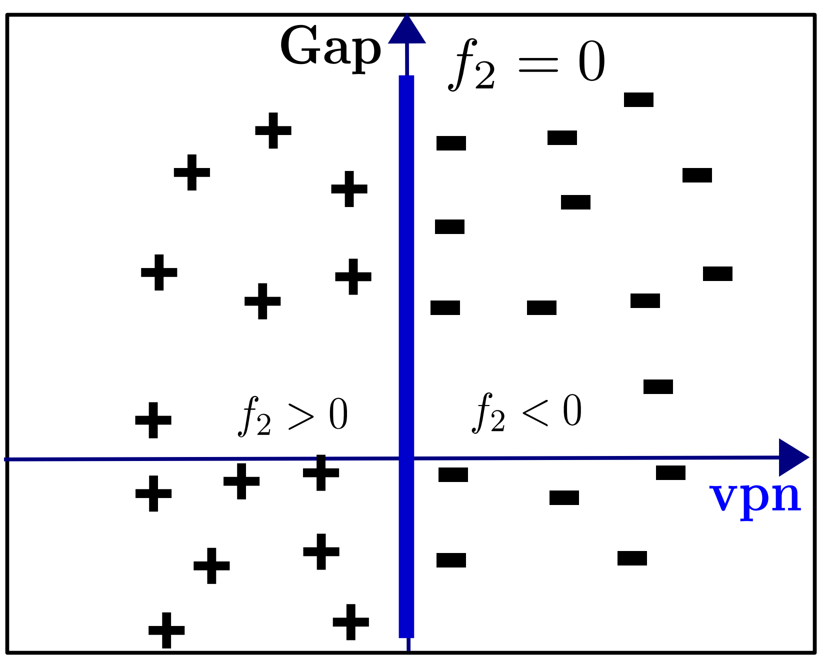


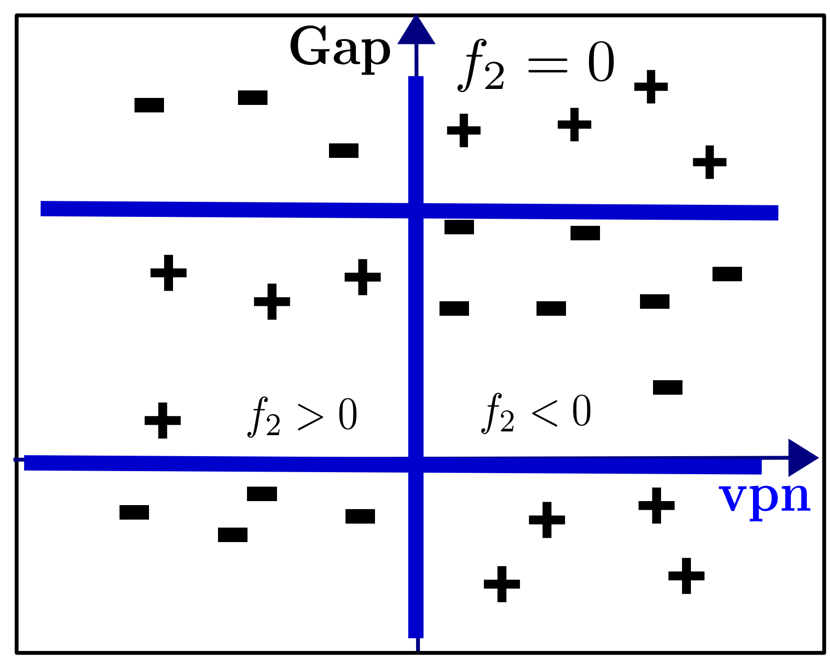
RRAM Model



clipping functions

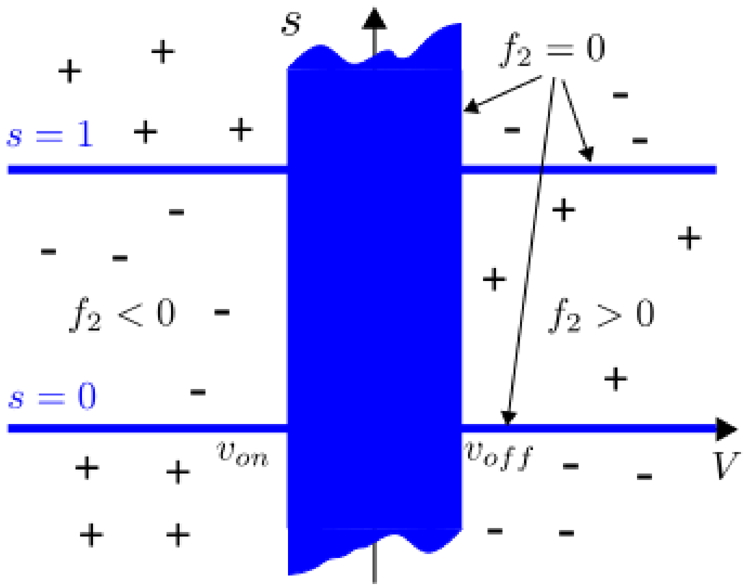
Analogy: MEMS switch
Zener diode voltage regulator



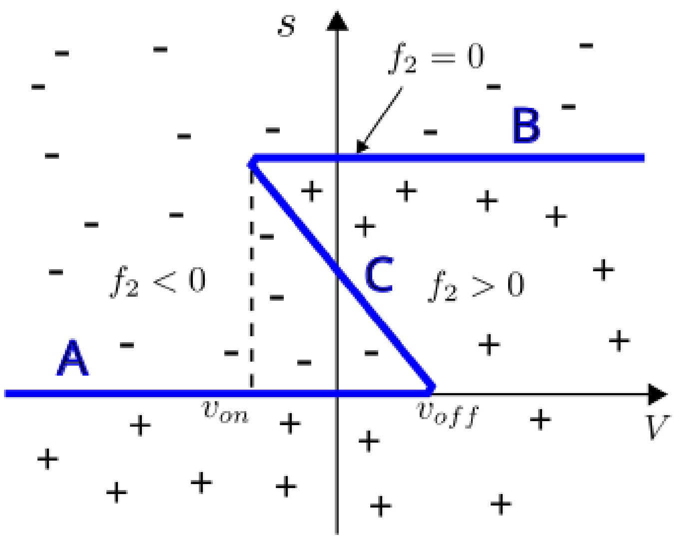


Memristor Models

Another (deeper) problem with f_2



TEAM/VTEAM, Yakopcic models



fix flat f_2 region

$$f_2 = \begin{cases} k_{off} \cdot (\frac{vpn}{v_{off}} - 1)^{\alpha_{off}}, & \text{if } vpn > v_{off} \\ k_{on} \cdot (\frac{vpn}{v_{on}} - 1)^{\alpha_{on}}, & \text{if } vpn < v_{on} \\ 0, & \text{otherwise} \end{cases}$$

$$f_2 = \begin{cases} k_{off} \cdot (\frac{vpn - v^*}{v_{off}})^{\alpha_{off}}, & \text{if } vpn > v^* \\ k_{on} \cdot (\frac{vpn - v^*}{v_{on}})^{\alpha_{on}}, & \text{otherwise,} \end{cases}$$

where

$$v^* = (1 - s) \cdot v_{off} + s \cdot v_{on}$$

Memristor Models

$$\frac{d}{dt}\mathbf{s} = f_2\left(\mathbf{vpn},\ \mathbf{s}\right)$$

Available f2:

1. linear ion drift

$$f_2 = \mu_v \cdot R_{on} \cdot f_1(\mathbf{vpn},\ s)$$

2. nonlinear ion drift

$$f_2 = a \cdot \mathbf{vpn}^m$$

3. Simmons tunnelling barrier

$$f_2 = \begin{cases} c_{off} \cdot \sinh(\frac{i}{i_{off}}) \cdot \exp(-\exp(\frac{s-a_{off}}{w_c} - \frac{i}{b}) - \frac{s}{w_c}), & \text{if } i \geq 0 \\ c_{on} \cdot \sinh(\frac{i}{i_{on}}) \cdot \exp(-\exp(\frac{a_{on}-s}{w_c} + \frac{i}{b}) - \frac{s}{w_c}), & \text{otherwise,} \end{cases}$$

4. TEAM model

5. Yakopcic model

6. Stanford/ASU

$$f_2 = -v_0 \cdot \exp(-\frac{E_a}{V_T}) \cdot \sinh(\frac{\mathbf{vpn} \cdot \gamma \cdot a_0}{t_{ox} \cdot V_T})$$

$$\mathbf{ipn} = f_1\left(\mathbf{vpn},\ \mathbf{s}\right)$$

Available f1:

1. $$f_1 = (R_{on} \cdot s + R_{off} \cdot (1-s))^{-1} \cdot \mathbf{vpn}$$

2. $$f_1 = \frac{1}{R_{on}} \cdot e^{-\lambda \cdot (1-s)} \cdot \mathbf{vpn}$$

3. $$f_1 = s^n \cdot \beta \cdot \sinh(\alpha \cdot \mathbf{vpn}) + \chi \cdot (\exp(\gamma \cdot) - 1)$$

4. $$\mathrm{f}_1 = \begin{cases} A_1 \cdot s \cdot \sinh(B \cdot \mathbf{vpn}), & \text{if } \mathbf{vpn} \geq \mathbf{0} \\ A_2 \cdot s \cdot \sinh(B \cdot \mathbf{vpn}), & \text{otherwise.} \end{cases}$$

5. $$f_1 = I_0 \cdot e^{-\mathbf{Gap}/g0} \cdot \sinh(\mathbf{vpn}/V_0)$$
$$\mathbf{Gap} = s \cdot minGap + (1-s) \cdot maxGap.$$

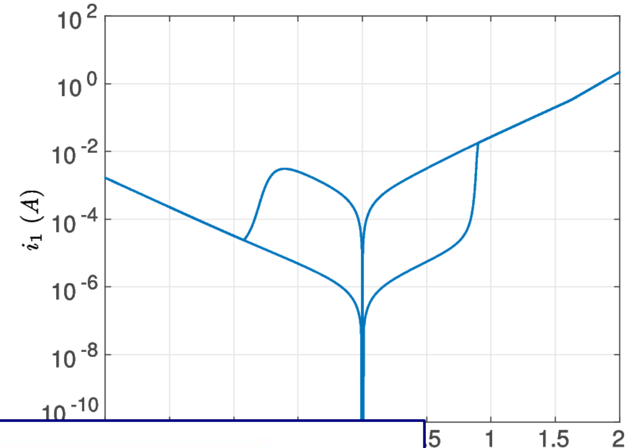
- **set up boundary**
- **fix f2 flat regions**
- **smooth, safe funcs, scaling, etc.**

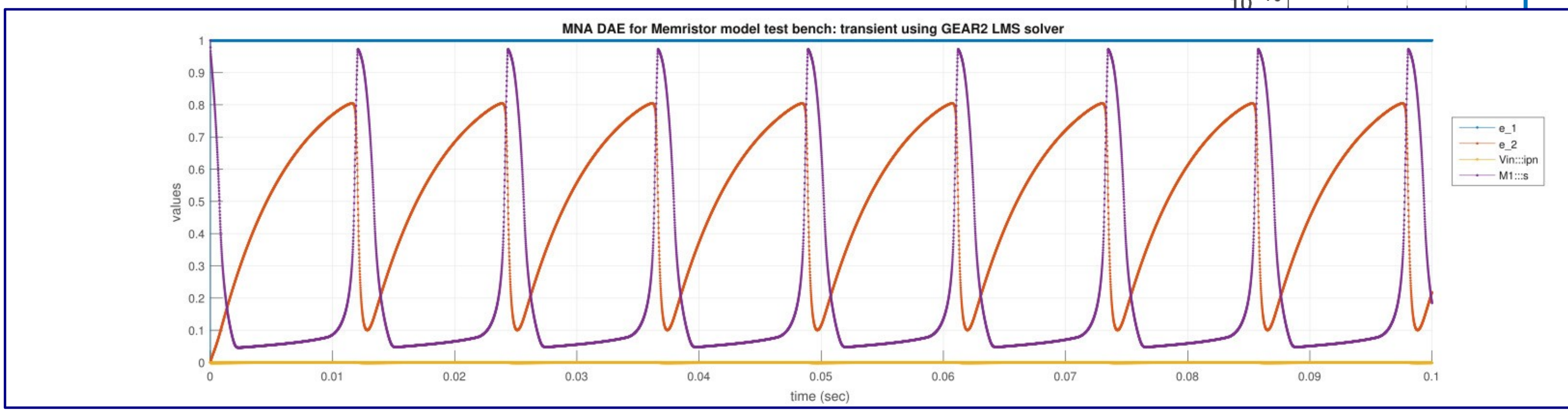
Wang/Roychowdhury. "Well-Posed Models of Memristive Devices." arXiv preprint (2016).

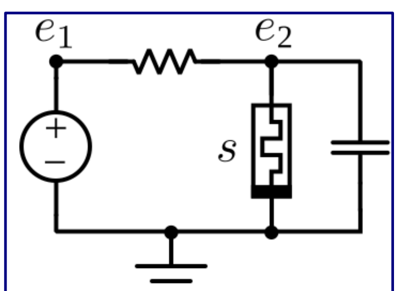
Memristor Models

A collection of 30 models:

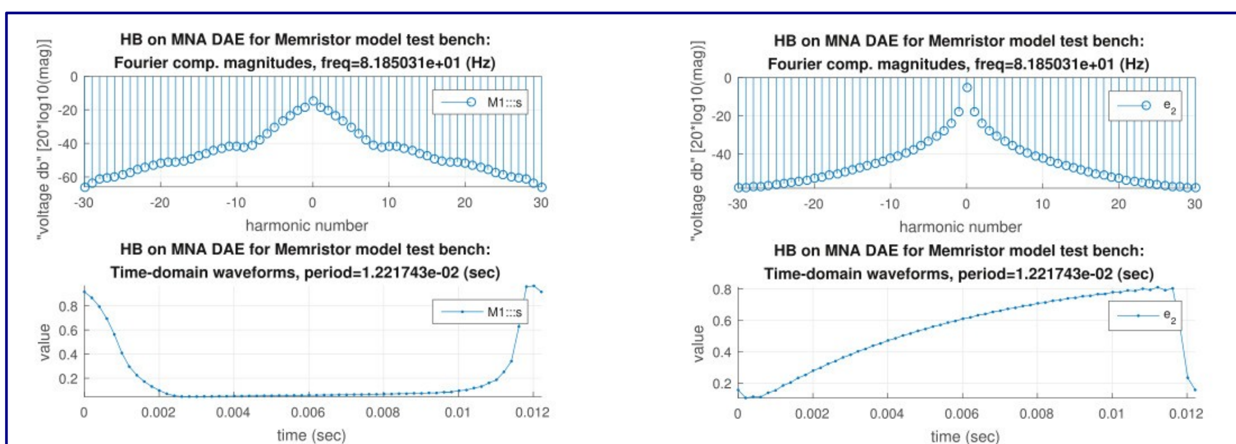
- all smooth, all well posed
- not just RRAM, but general memristive devices
- not just bipolar, but unipolar
- not just DC, AC, TRAN, but homotopy, PSS, ...



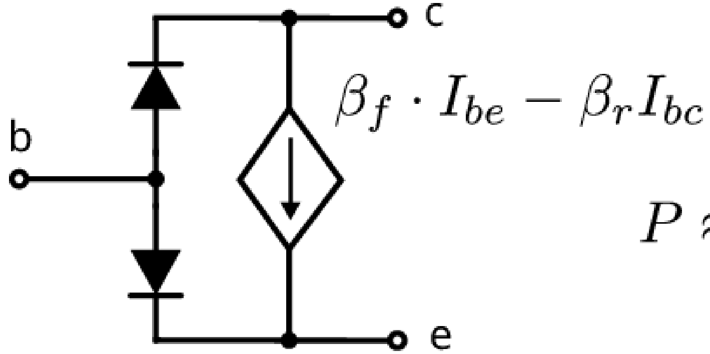


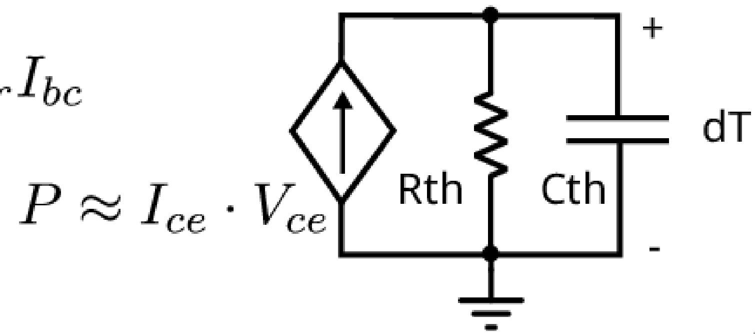


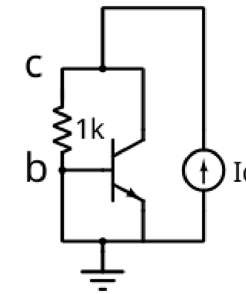
PSS using HB



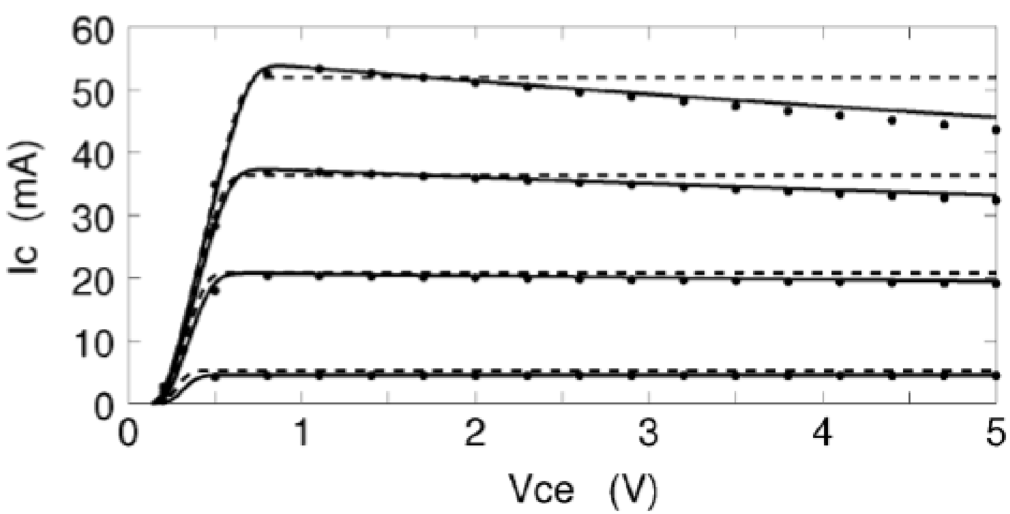
HBT with Thermal Effects



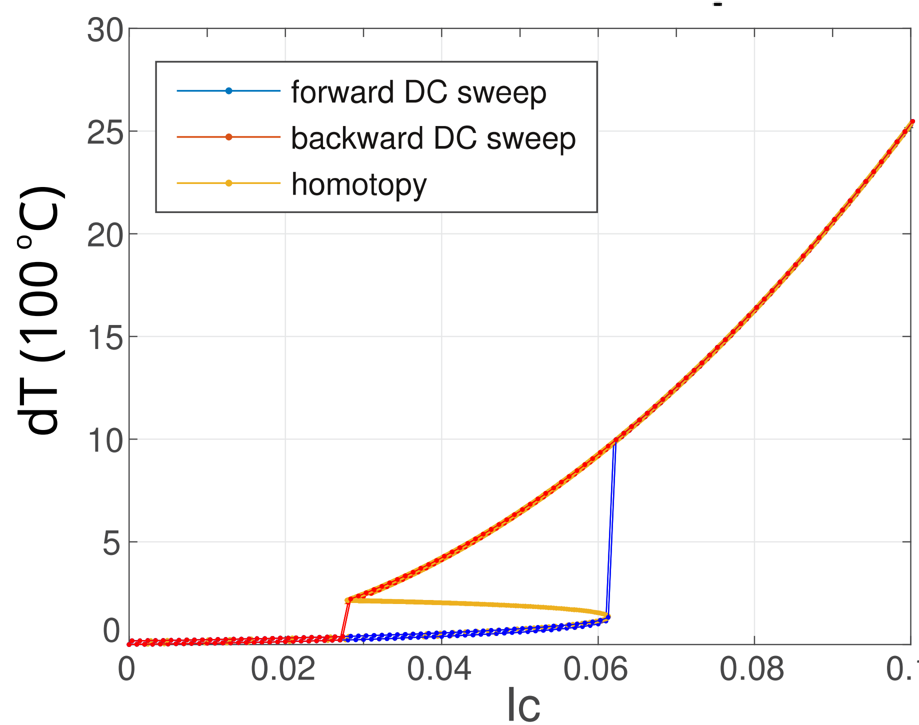




$$\beta_f = \beta_{f0} - d\beta_f \cdot dT$$

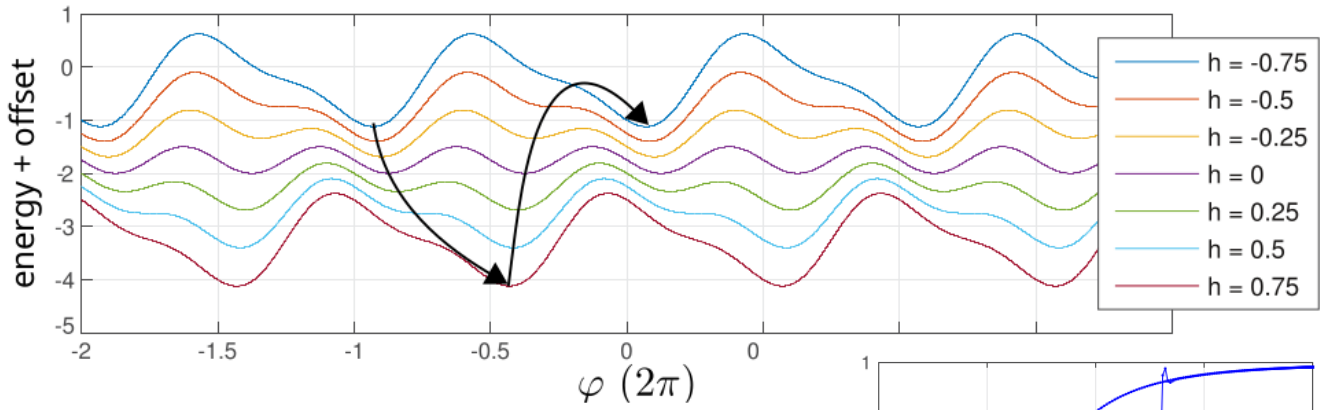


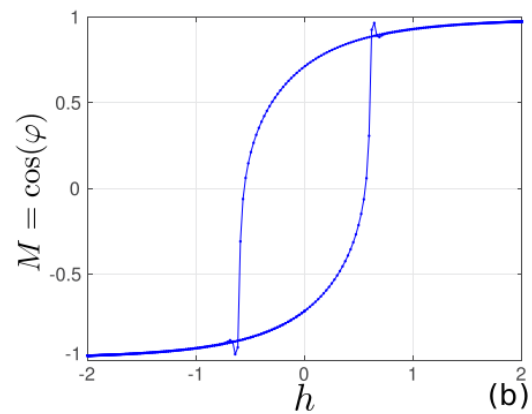
Rudolph "Uniqueness problems in compact HBT models caused by thermal effects." IEEE Trans. MTT 2004



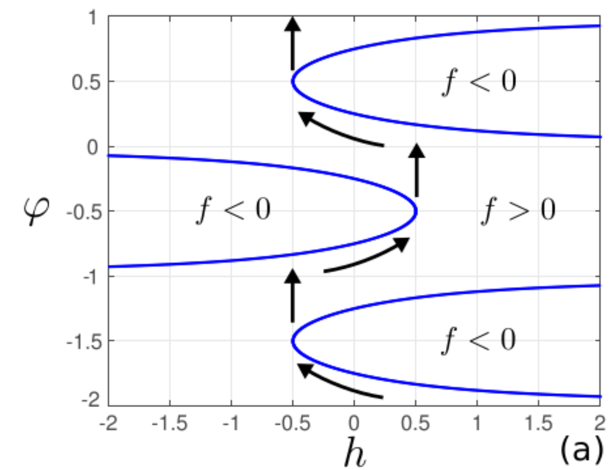
Ferromagnet Models

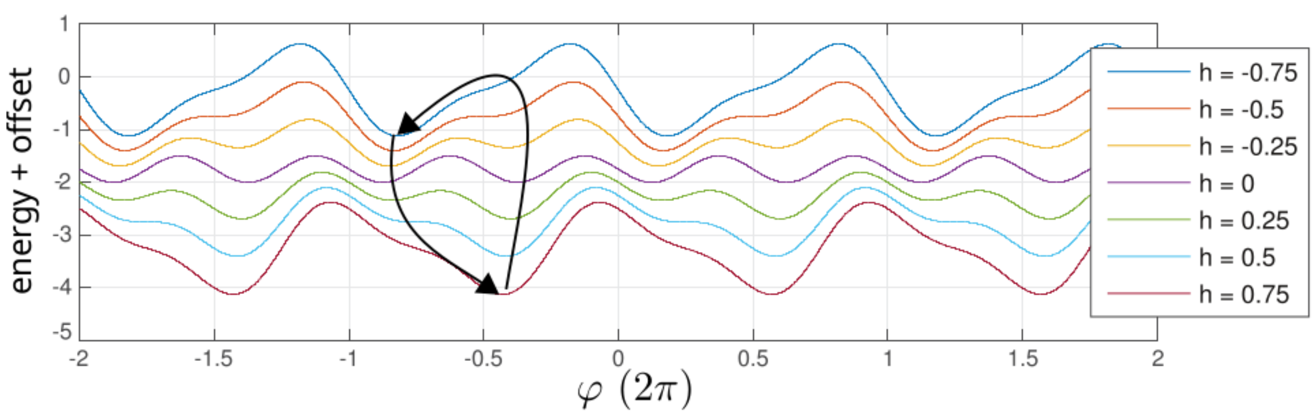
Stoner-Wohlfarth Model: $E(h,\varphi) = \frac{1}{4} - \frac{1}{4}\cos(2(\varphi - \theta)) - h\cos(\varphi)$

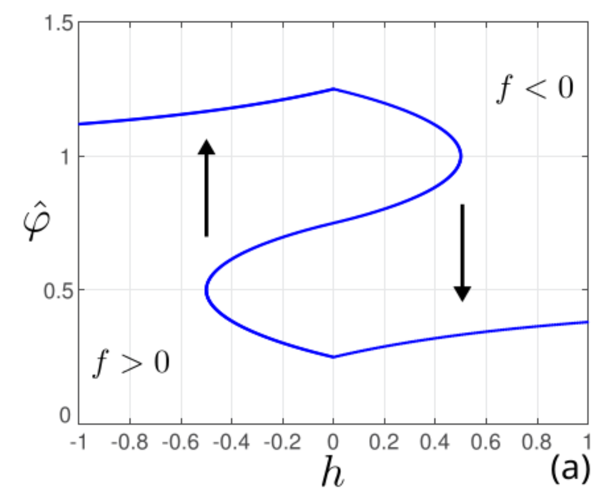




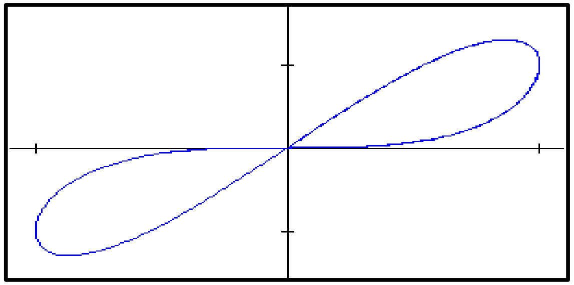
$M = \cos(\varphi)$







How to model hysteresis?

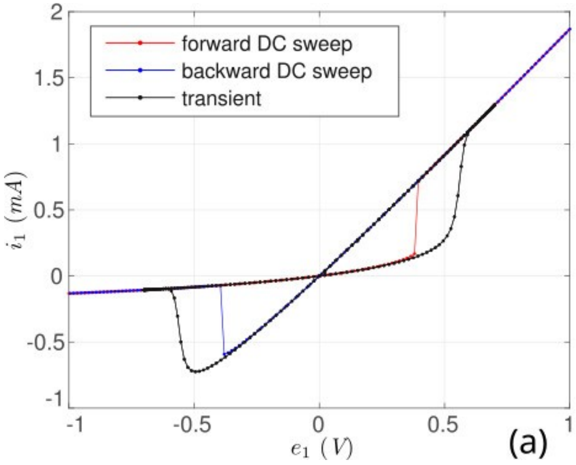


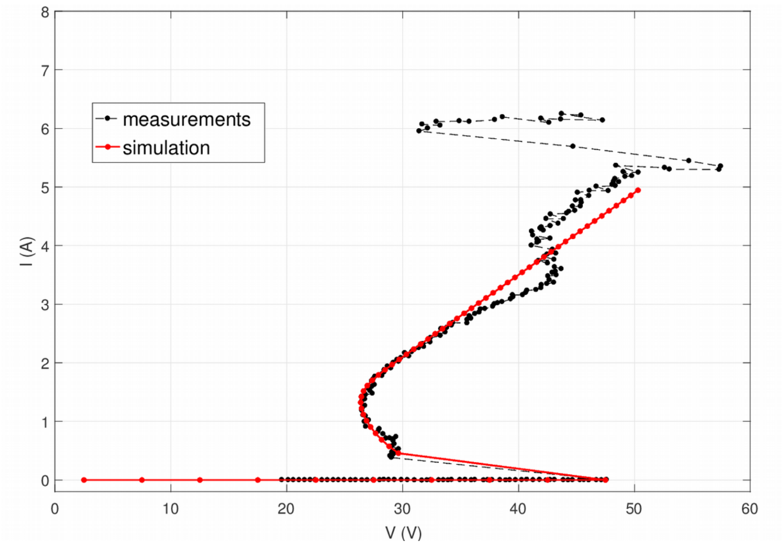
- hysteresis ≠ discontinuity or if-else
- hysteresis ≠ use $abstime
  - or @(initial_step), $bound_step, *etc.*
- hysteresis ≠ hybrid models
- hysteresis/multistability ≠ "flat" regions w zero derivatives

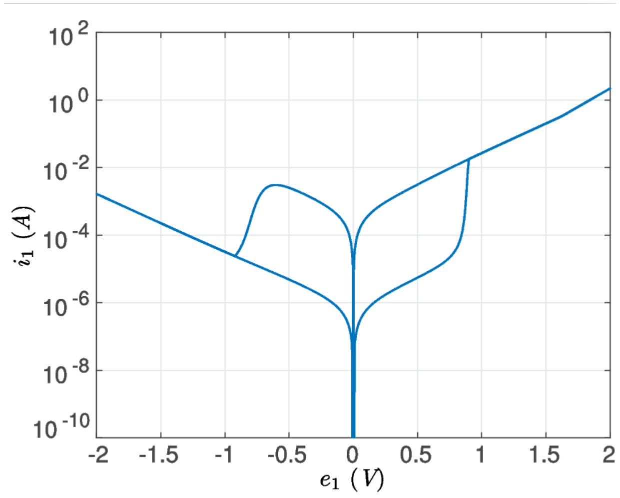
⇕

- model hysteresis using internal state variable
  - proper design of dynamics
- write internal unknown in Verilog-A
  - use potentials/flows
- set bounds for internal unknown with equations
  - physical distance
  - clipping functions
- smoothness, continuity, finite precision issues, …

How to model hysteresis?







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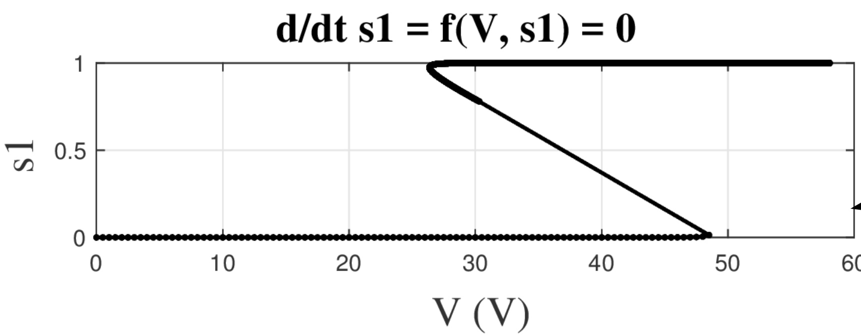
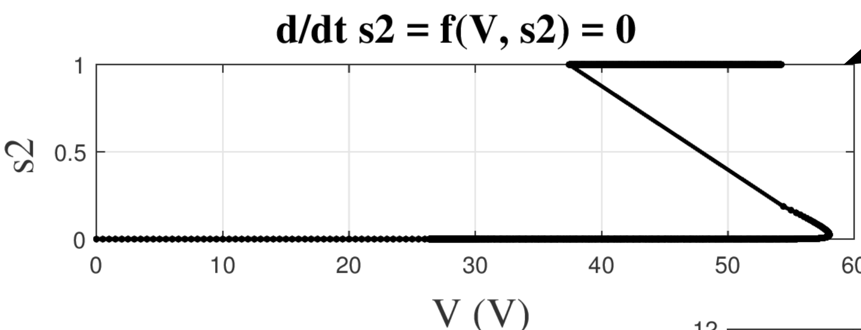
Modeling Second Snapback



$$I = I_{off} + s_1 \cdot I_{on1} + s_2 \cdot I_{on2}$$

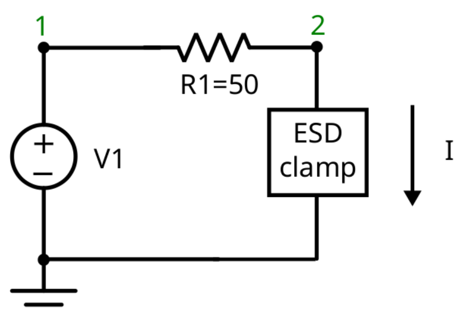
another internal unknown

different transition voltages

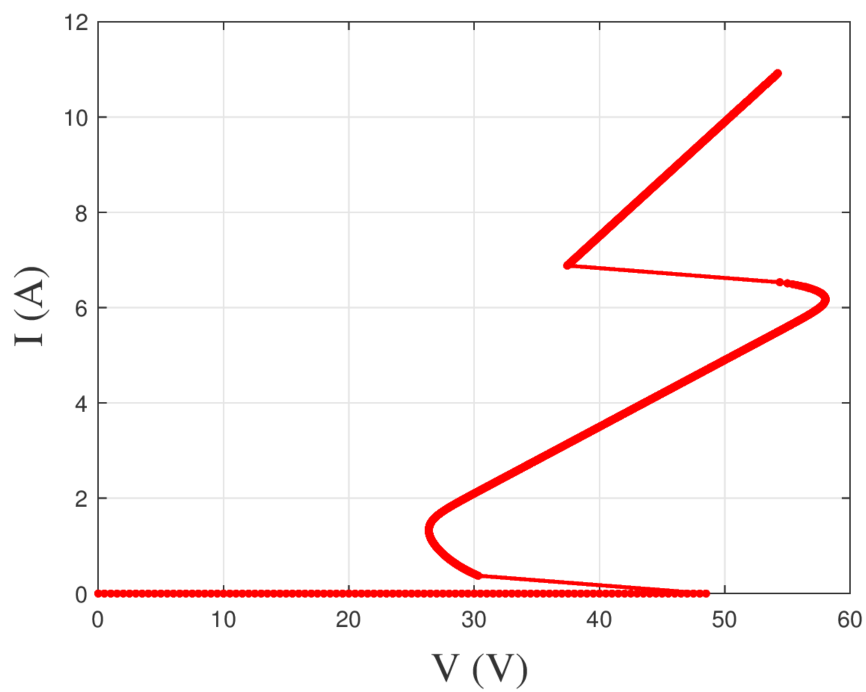



```
...
I(ns2,n) <+ (tanh(K2*(V2star + s2star)) - s2star)
                * smoothstep(s1-0.9, smoothing);
I(ns2,n) <+ ddt(-tau2*s2);
...
```

s2 dynamics is on only when s1 close to 1



simple TLP equivalent circuit



T. Wang, UC Berkeley

ESD Snapback Model