Does It Take Longer to Injection Lock a High-Q Oscillator?

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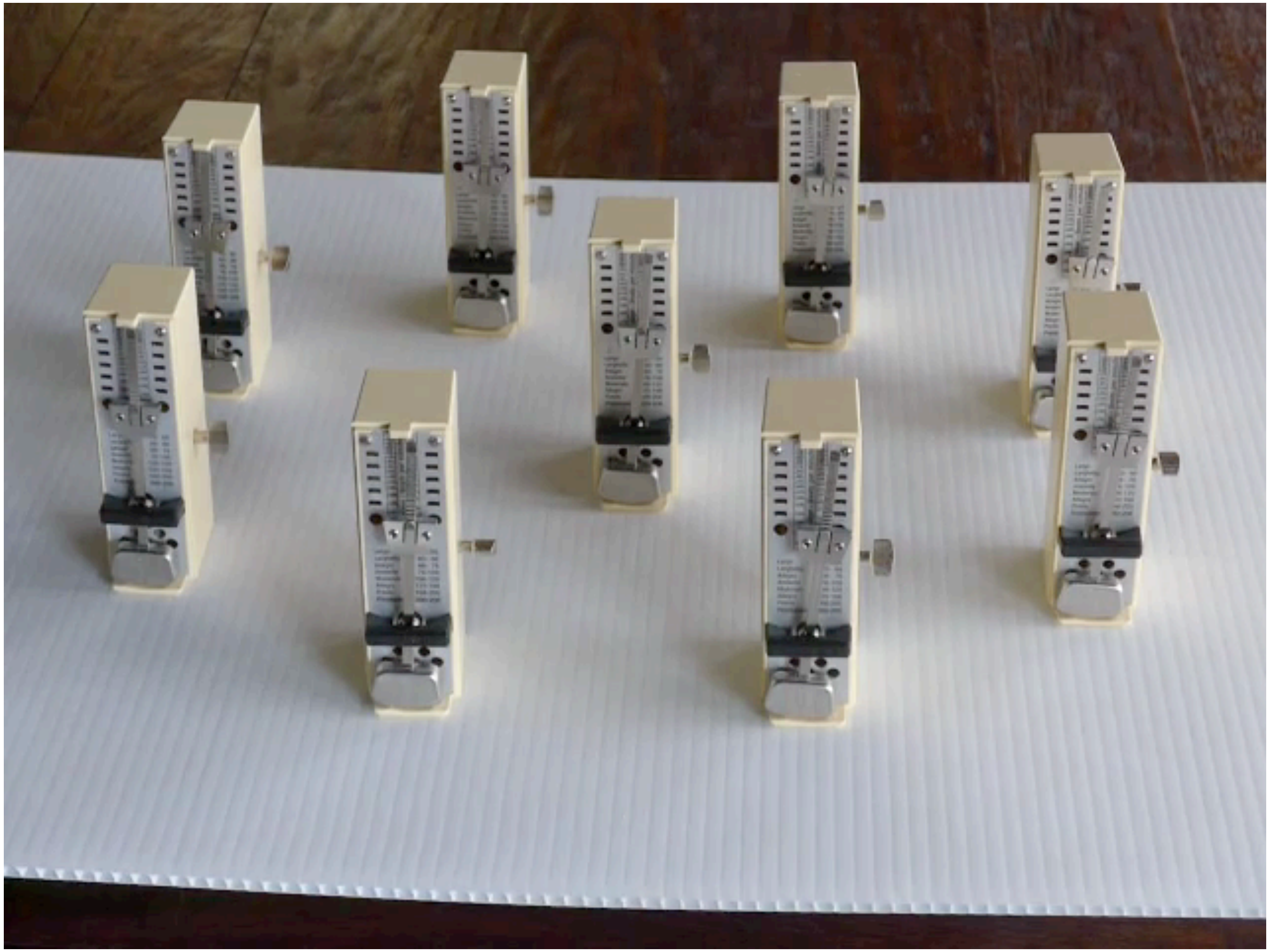
Injection Locking

- Oscillators can synchronize in phase/frequency



Injection Locking

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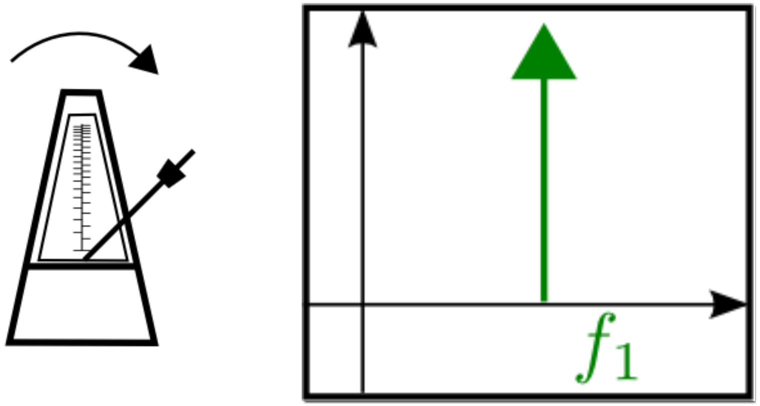
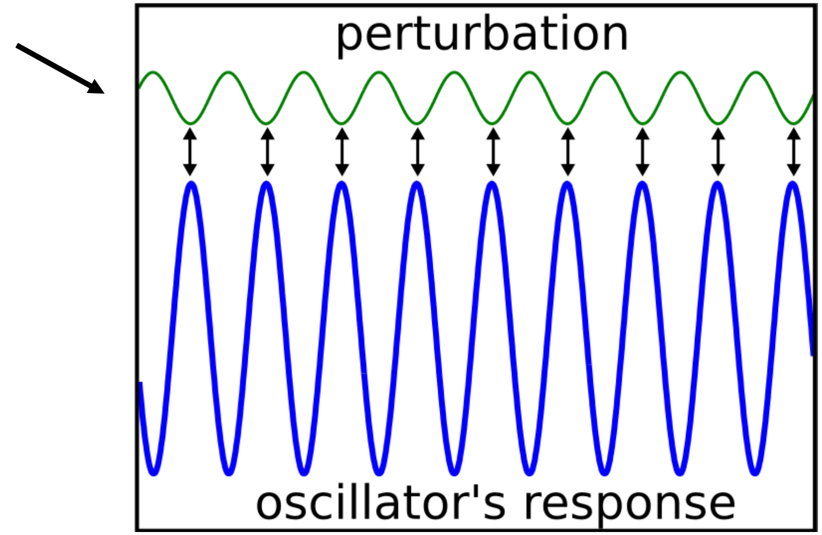
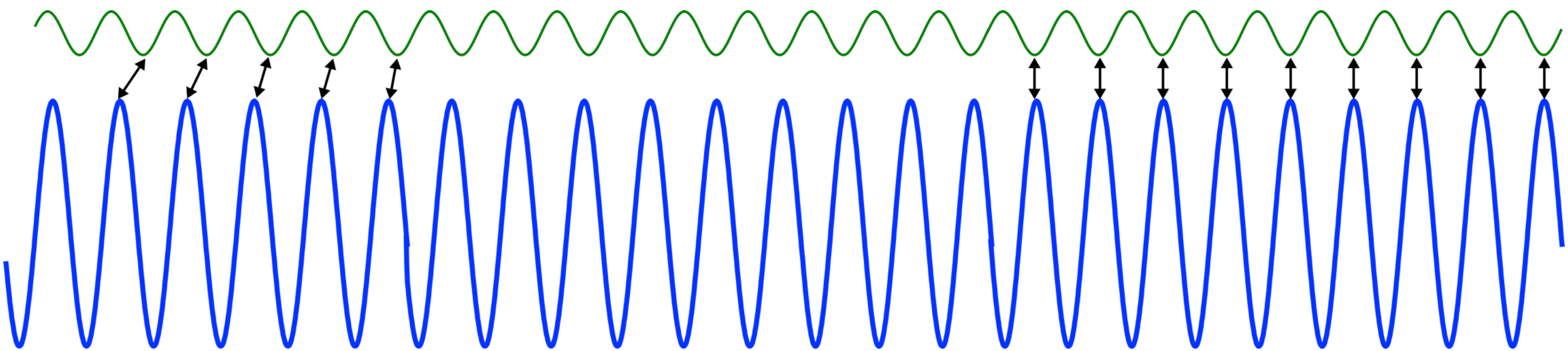


phase lock

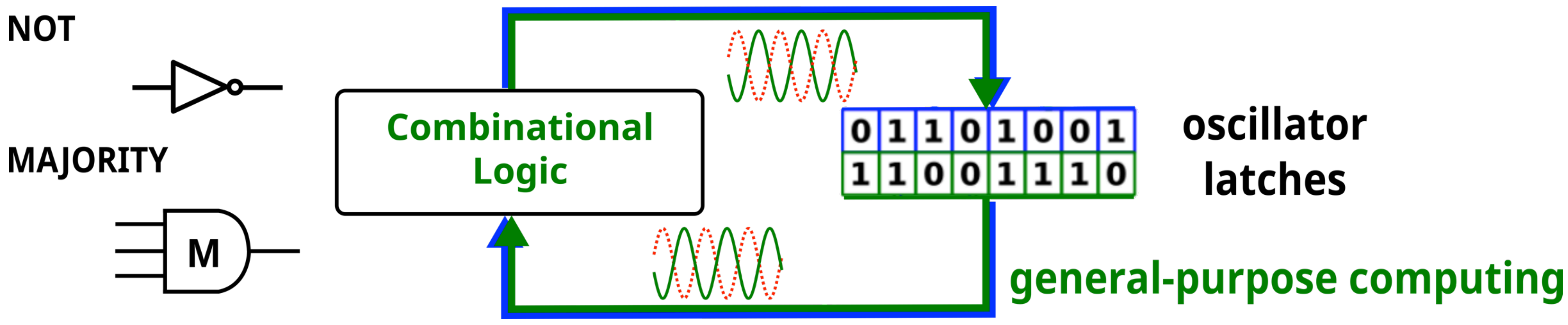




How fast is injection locking?

Does It Take Longer to Injection Lock a High-Q Oscillator?

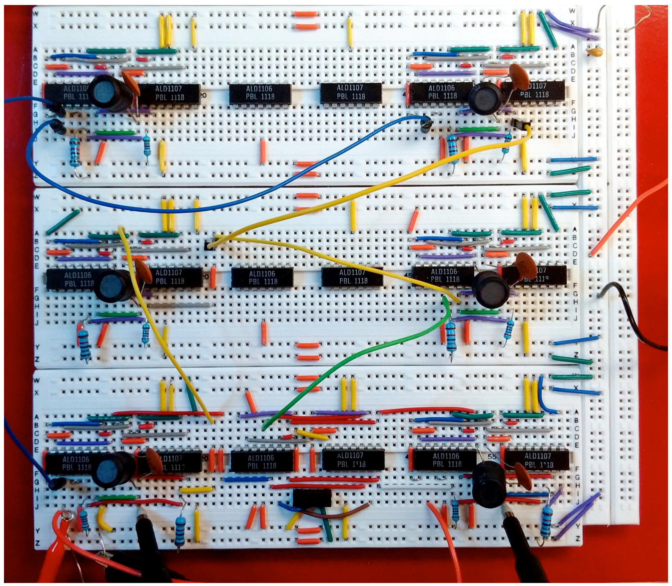
- *Why do we ask?*
  - Applications of IL:
    - quadrature oscillators
    - injection-locked PLLs
    - frequency dividers
    - optical lasers
    - ***Oscillator-based Boolean Computation***

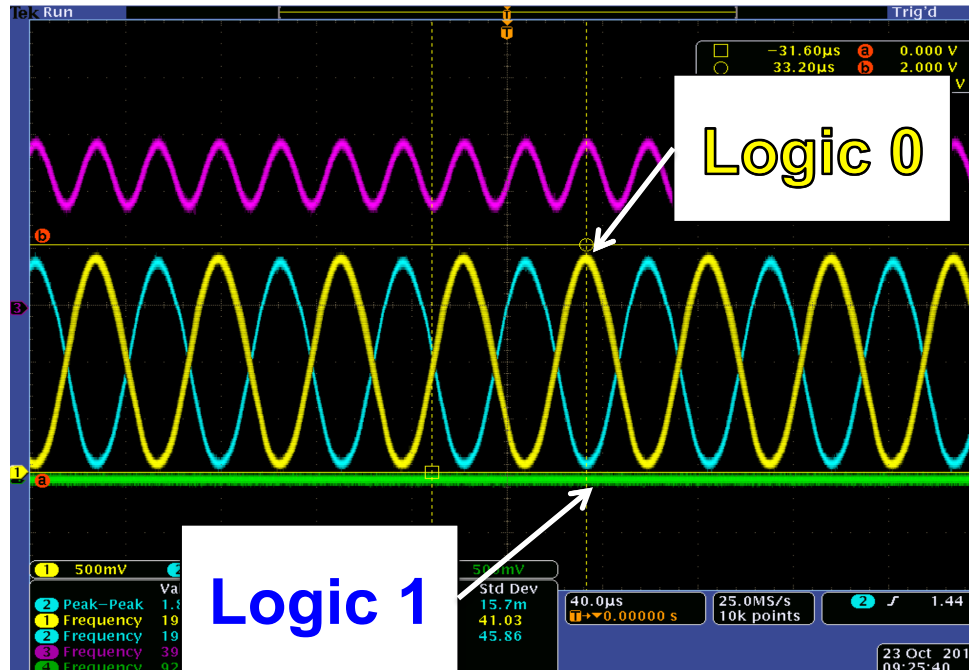


details: T. Wang, J. Roychowdhury, "PHLOGON: Phase-based LOGic using Oscillatory Nano-systems". Unconventional Computation & Natural Computation, 2014.

Does It Take Longer to Injection Lock a High-Q Oscillator?

- ***Oscillator-based Boolean Computation***

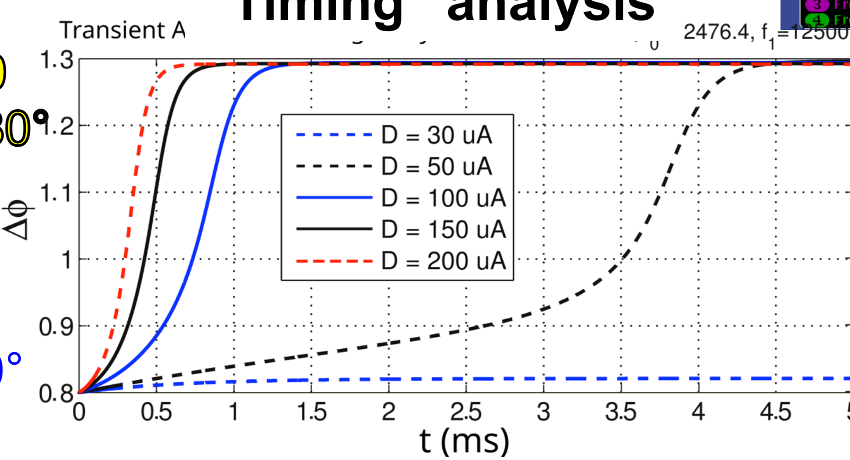




"Timing" analysis

logic 0
phase 180°

logic 1
phase 0°



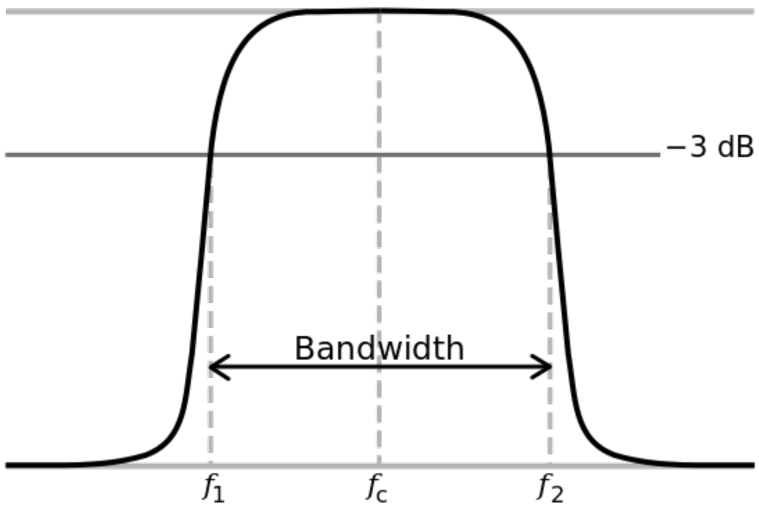
Speed vs. Power

how fast is injection locking → Speed

Q factor → Power

Q factor of an osc.: Definitions & Confusions

- $Q \stackrel{\text{def}}{=} \dfrac{f_r}{\Delta f}$



linear resonators only

high-Q resonator ≠ high-Q osc.

- $Q \stackrel{\text{def}}{=} 2\pi f_r \times \dfrac{\text{Energy Stored}}{\text{Power Loss}}$

damping systems

how to measure/characterize?

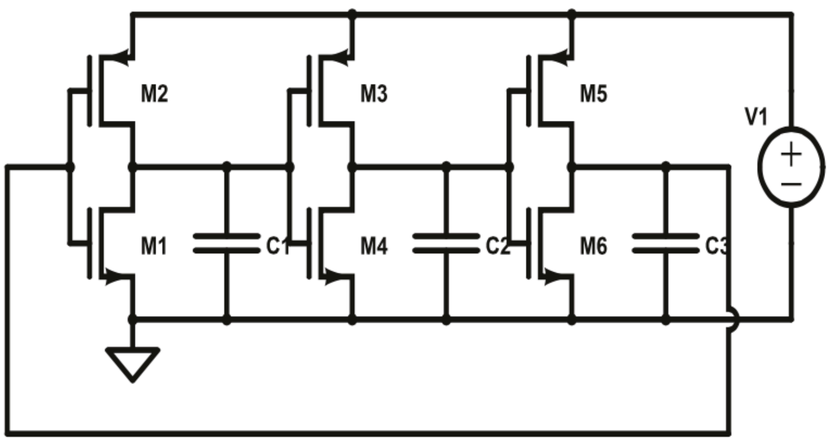
- $H(s) = \dfrac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$

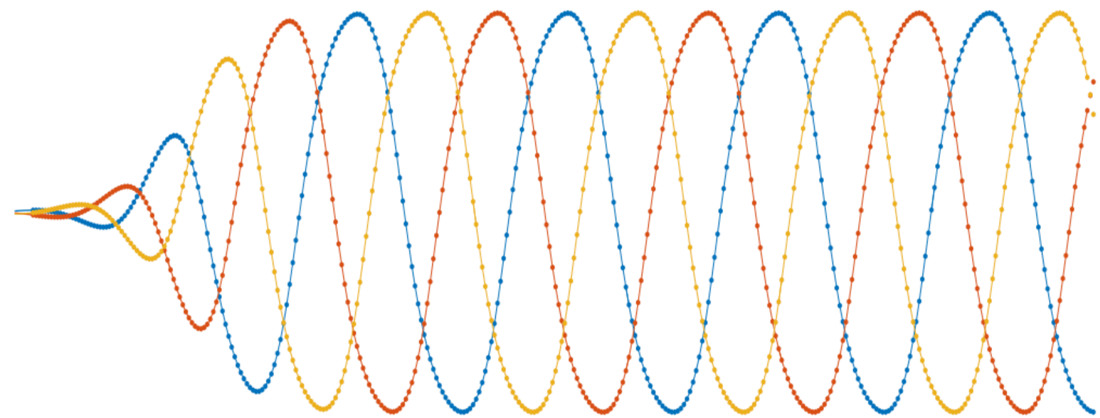
second order linear

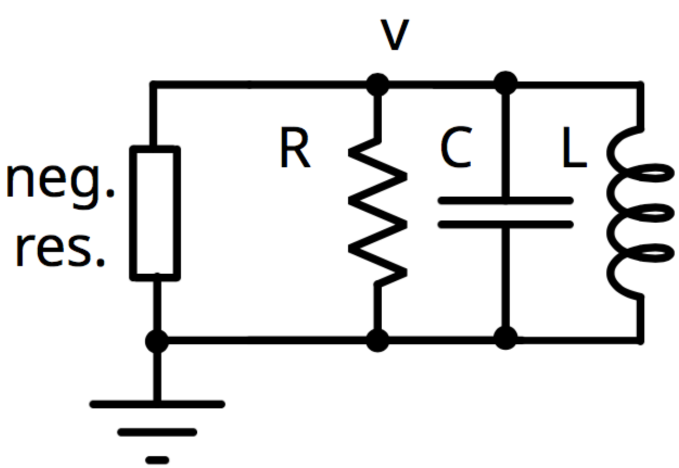
- $Q = R\sqrt{\dfrac{C}{L}} = \dfrac{R}{\omega_0 L} = \omega_0 RC$

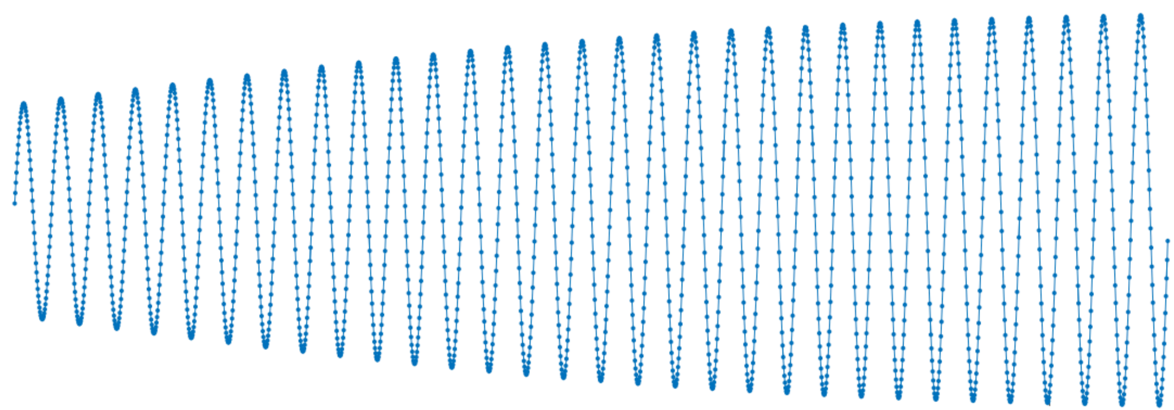
RLC only

Q factor of an oscillator: Intuition





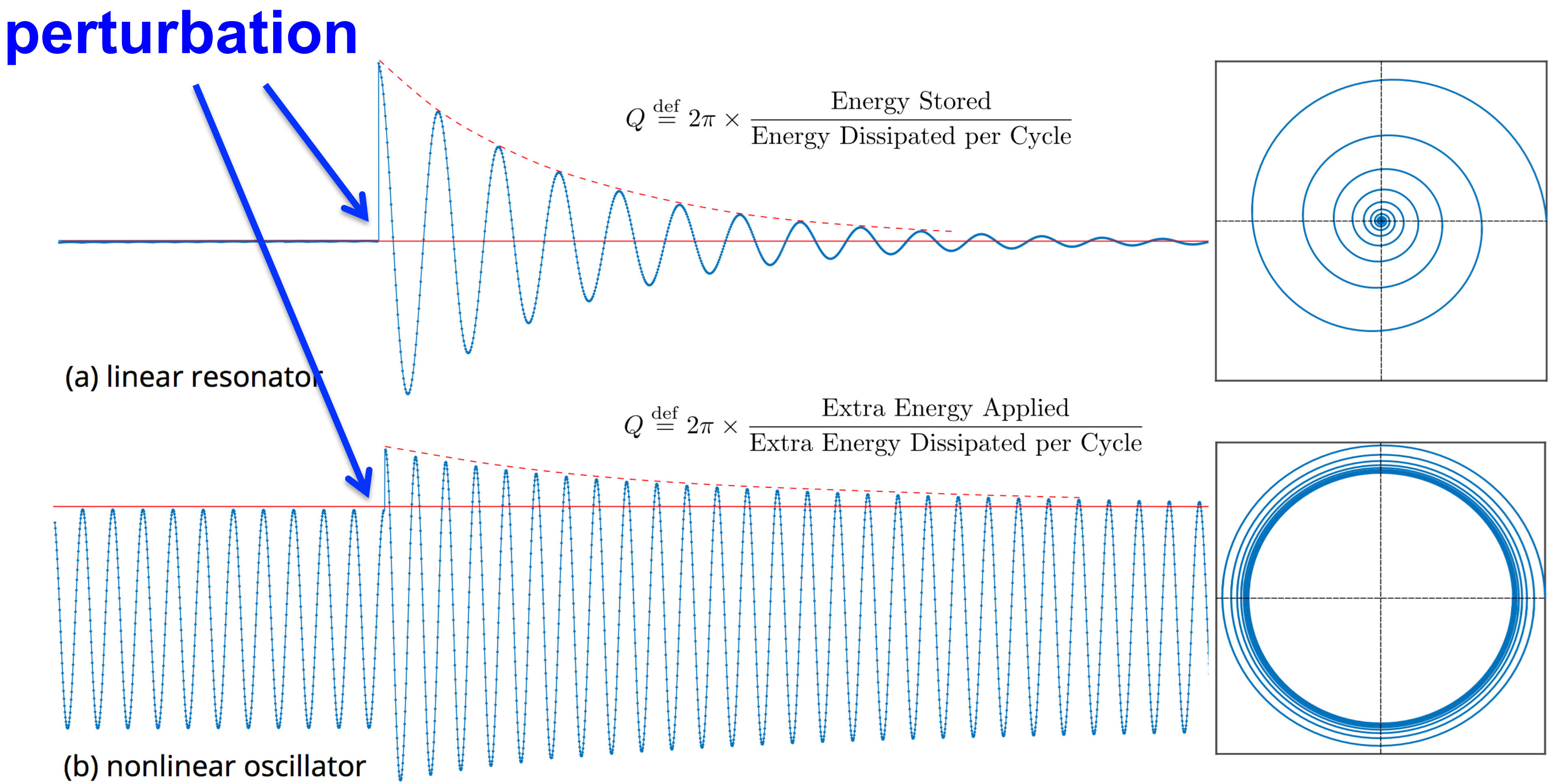




high-Q oscillators settle more slowly

(in amplitude)

Q factor of an oscillator: Our Definition

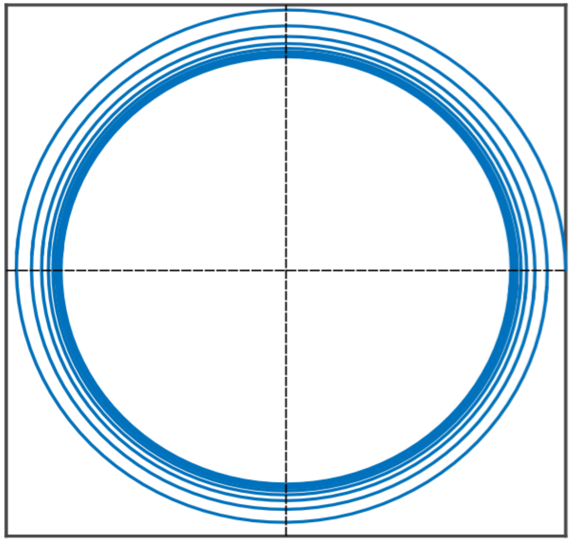


can be measured **not specific to osc. types**

Q factor: Mathematical Characterization

osc. DAE:

$$\frac{d}{dt}\vec{q}(\vec{x}(t)) + \vec{f}(\vec{x}(t)) + \vec{b}(t) = \vec{0}$$



Periodic Steady State (PSS)

$$\vec{x}_s(t) \qquad\qquad \vec{x}_s(t) = \vec{x}_s(t+T)$$

apply perturbation:

$$\vec{x}(t) = \vec{x}_s(t) + \Delta\vec{x}(t)$$

$$\frac{d}{dt}\vec{q}(\vec{x}_s(t) + \Delta\vec{x}(t)) + \vec{f}(\vec{x}_s(t) + \Delta\vec{x}(t)) = \vec{0}$$

Linear Periodically Time-Varying (LPTV) system:

$$\frac{d}{dt}\mathbf{C}(t)\cdot\Delta\vec{x}(t) + \mathbf{G}(t)\cdot\Delta\vec{x}(t) = \vec{0} \qquad \mathbf{C}(t) = \left.\frac{d\vec{q}}{d\vec{x}}\right|_{\vec{x}_s(t)} \qquad \mathbf{G}(t) = \left.\frac{d\vec{f}}{d\vec{x}}\right|_{\vec{x}_s(t)}$$

Q factor: Mathematical Characterization

Linear Periodically Time-Varying (LPTV) system:

$$\frac{d}{dt}\mathbf{C}(t)\cdot\Delta\vec{x}(t)+\mathbf{G}(t)\cdot\Delta\vec{x}(t)=\vec{0}$$

Fundamental Matrix of LPTV: $\mathbf{X}(t)$

$$\frac{d}{dt}\mathbf{C}(t)\cdot\mathbf{X}(t)+\mathbf{G}(t)\cdot\mathbf{X}(t)=\vec{0}$$

$$\mathbf{X}(0)=\mathbf{I_n}$$

$\Delta\vec{x}(T)=\mathbf{X}(T)\cdot\Delta\vec{x}(0)$ ⟵ $\mathbf{X}(T)$ determines $\Delta\vec{x}(0)\rightarrow\Delta\vec{x}(T)$

Eigenanalysis on $\mathbf{X}(T)$

- $\lambda_{\max}=\lambda_1=1$
- λ_2 characterizes the decay of amplitude!

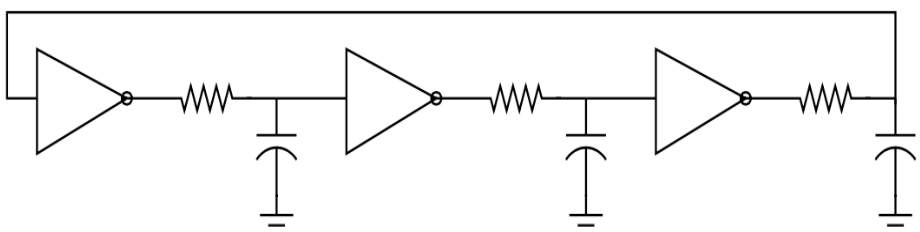
Q factor: Mathematical Characterization

Eigenanalysis on $\mathbf{X}(T)$

- $\lambda_{\max} = \lambda_1 = 1$
- λ_2 characterizes the decay of amplitude!

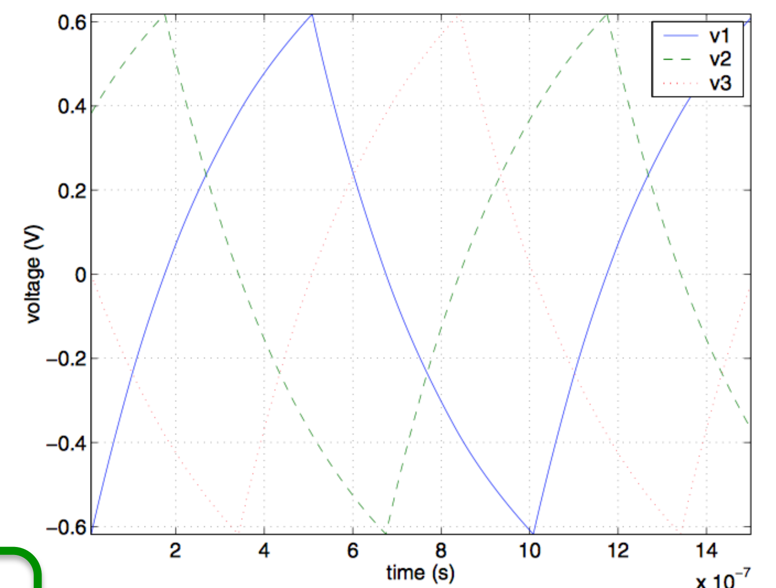
$\lambda_2^Q < 0.05$ means: after Q cycles, magnitude drops below 5%

An analytical example:



$$f(v) = \begin{cases} -A, & if\ v > 0 \\ A, & otherwise. \end{cases}$$

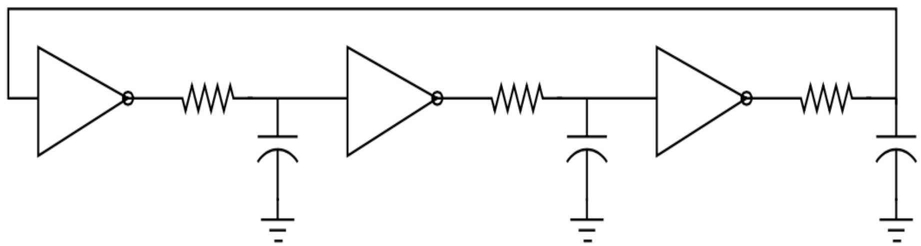
ideal ring osc.

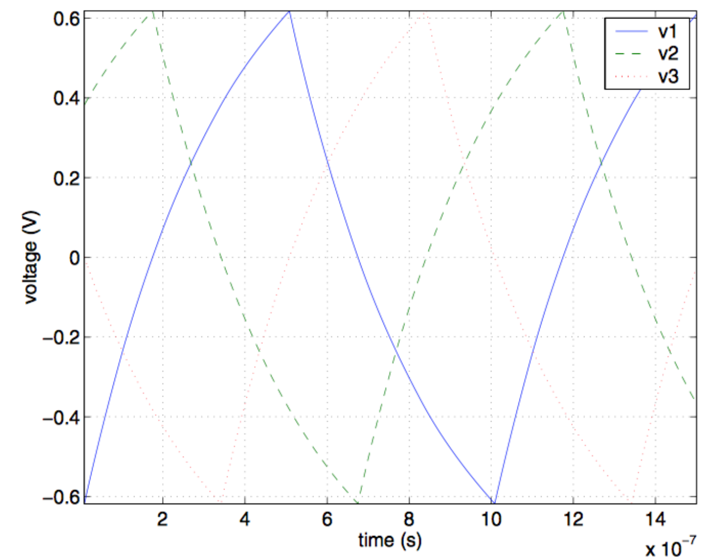


$$\lambda_2 = \left(\frac{\sqrt{5}-1}{2}\right)^6 \approx 0.0557$$ → $$Q \approx 1.1$$

details: Srivastava/Roychowdhury, “Analytical Equations for Nonlinear Phase Errors and Jitter in Ring Oscillators”, TCAS I, 2007.

Q factor: Numerical Results





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ideal ring osc.

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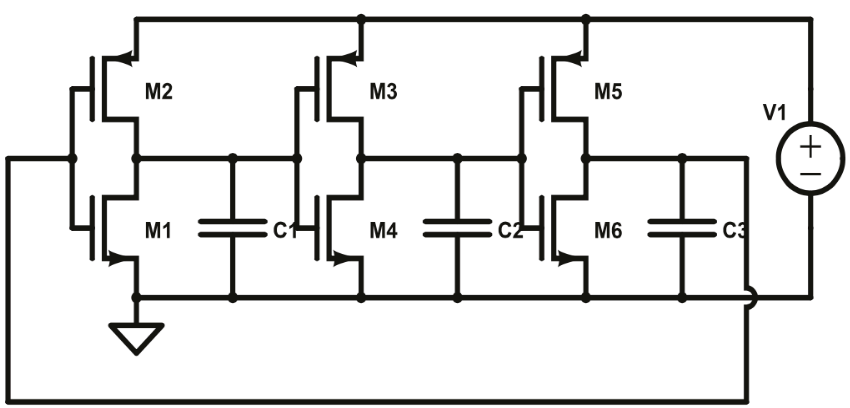
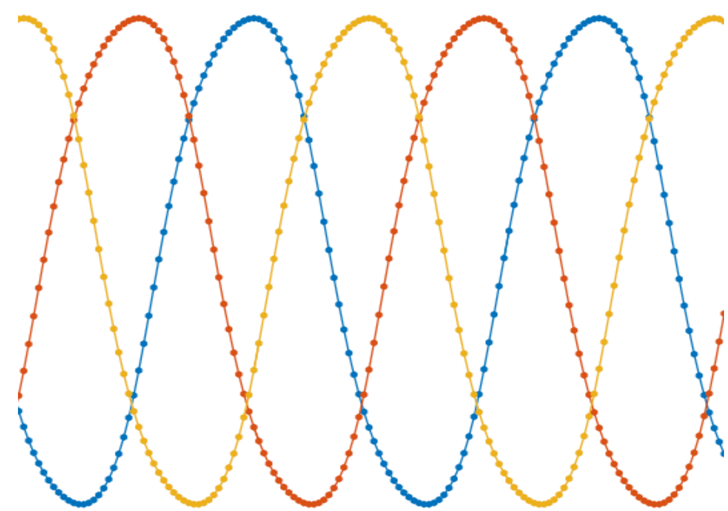
"realistic" ring osc.

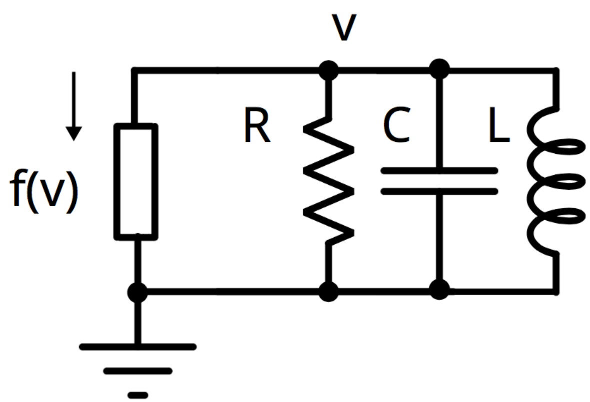


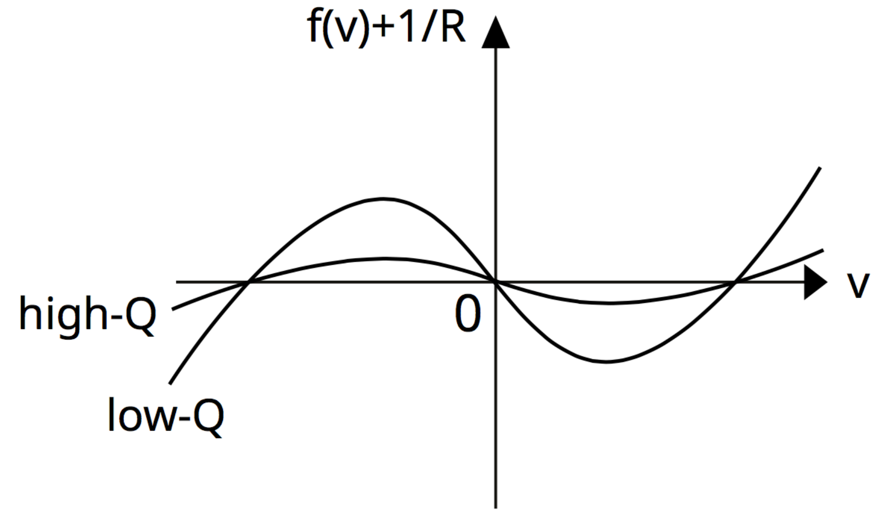


$$\lambda_2 \approx 0.067 \rightarrow Q \approx 1$$

Q factor: Numerical Results

LC osc.





$$f(v) + \frac{1}{R} = K \cdot (v - \tanh(1.01 \cdot v))$$

K = 20 (low-Q): $\lambda_2 \approx 0.11$ → $Q \approx 1.4$

K = 1 (high-Q): $\lambda_2 \approx 0.83$ → $Q \approx 16$

Q factor: Numerical Results

a chemical reaction osc.

Soloveichik's chemical reaction osc.

```
A + B = 2B
B + C = 2C
C + A = 2A
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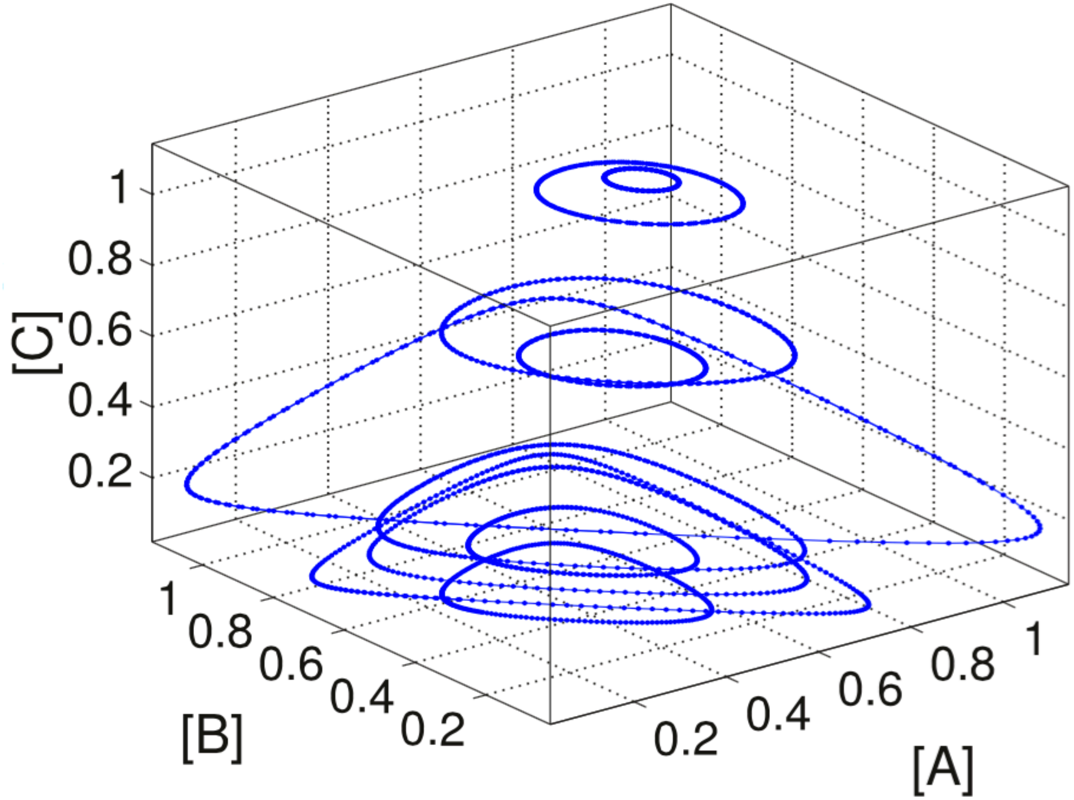
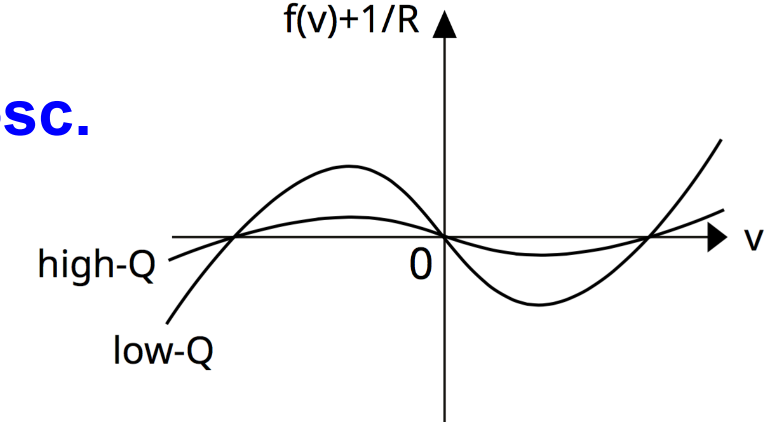


$$\lambda_2 = 1$$ → **not amplitude-stable**

High-Q oscillators settle more slowly in amplitude

LC osc.



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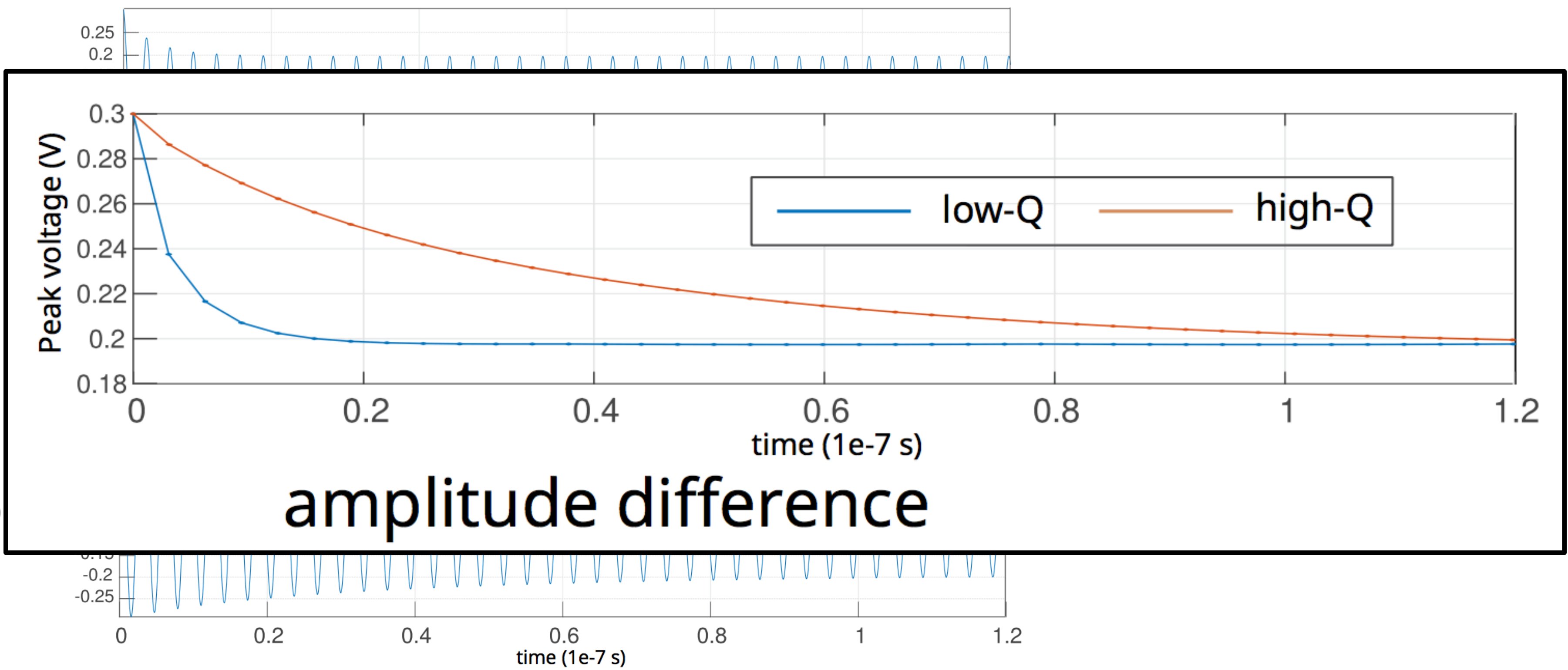
K = 20 (low-Q)

K = 1 (high-Q)



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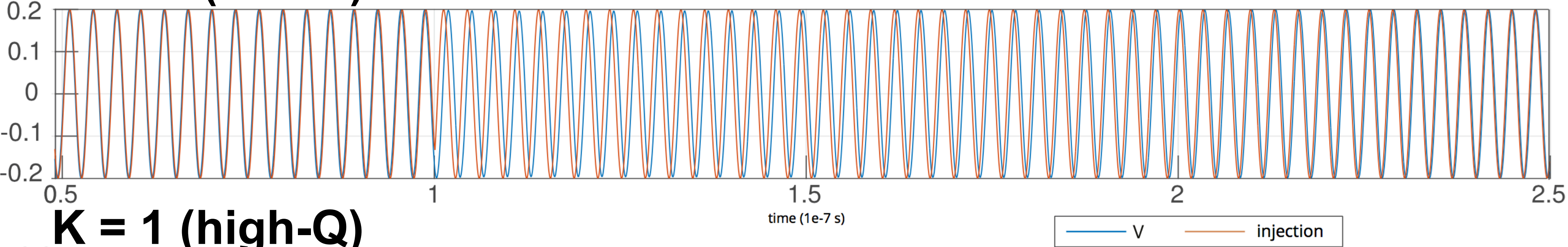
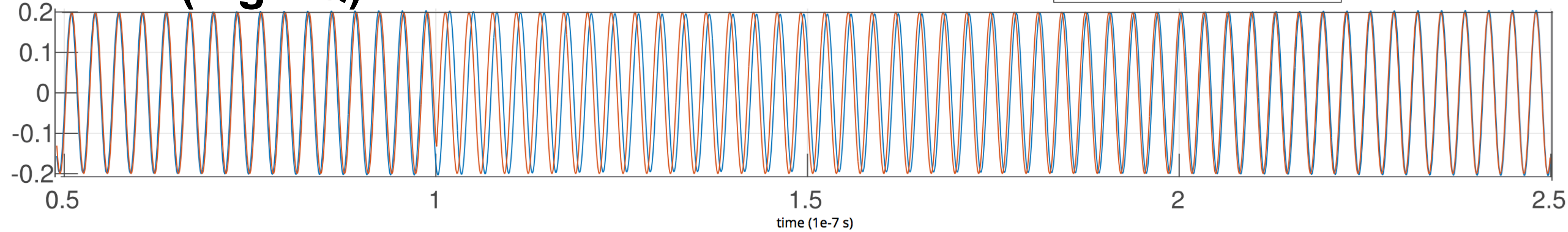
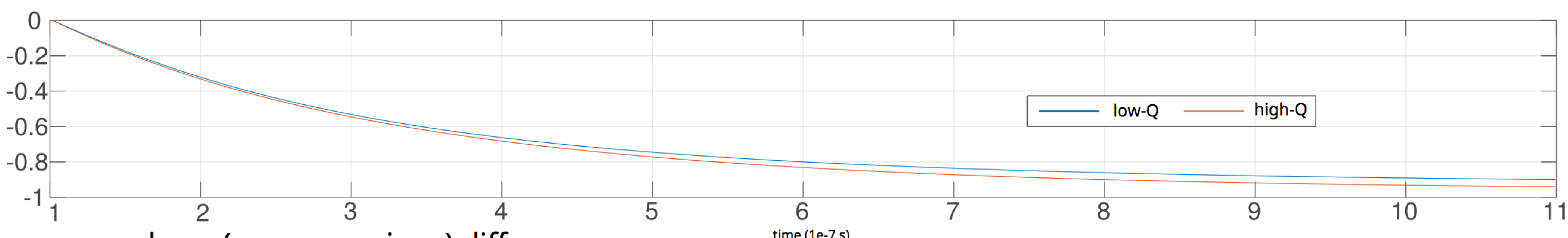
K = 20 (low-Q)



K = 1 (high-Q)





phase (zeros crossings) difference

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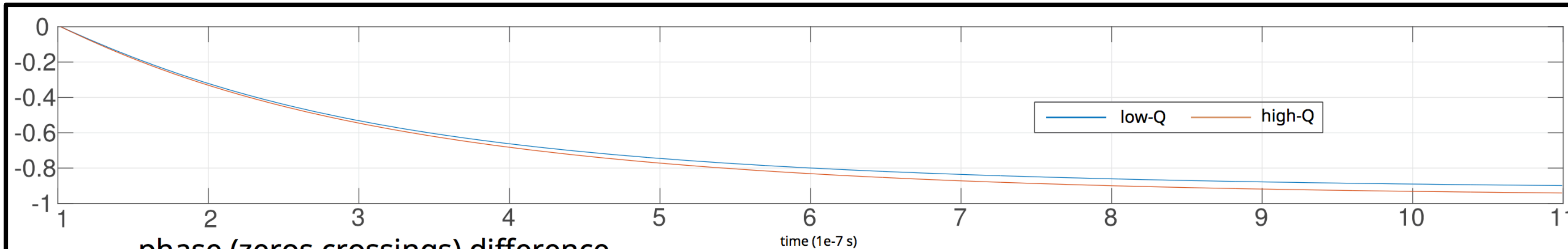

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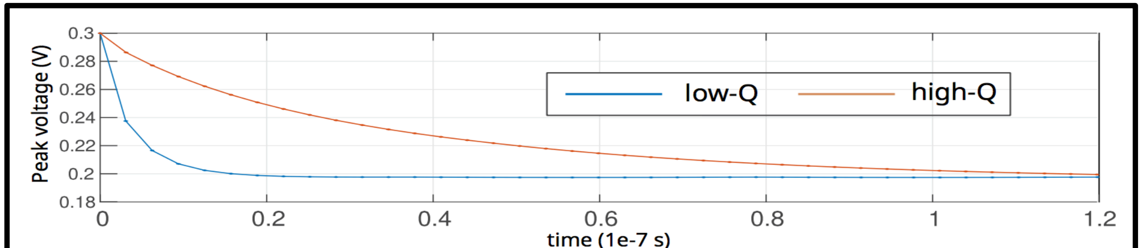


amplitude difference

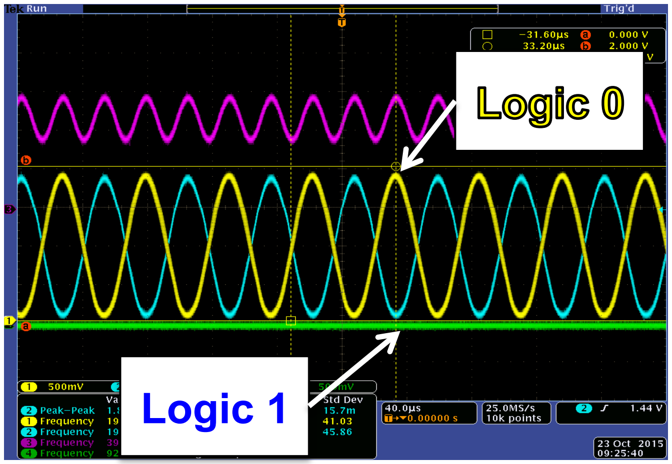
“decoupled” phase and amplitude settling behaviours

loose explanation:

$$Q \Leftrightarrow \lambda_2 \text{ of } \mathbf{X}(T)$$

$$\text{phase-macromodel} \Leftrightarrow \vec{v}_1(t) \text{ corresponding to } \lambda_1 \text{ of } \mathbf{X}(T)$$

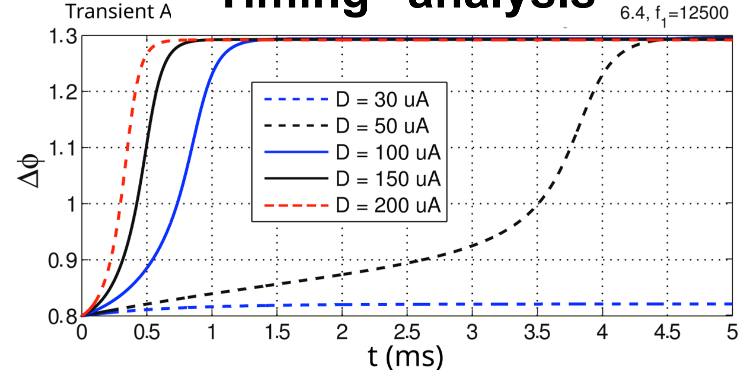
Summary

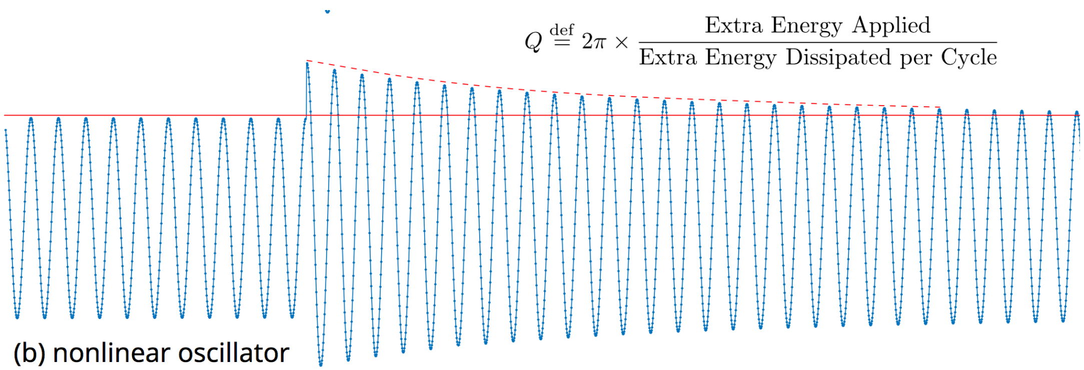


Speed vs. Power

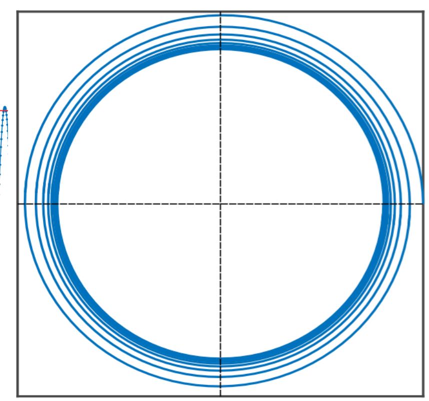
how fast is injection locking ↗ ↖ Q factor

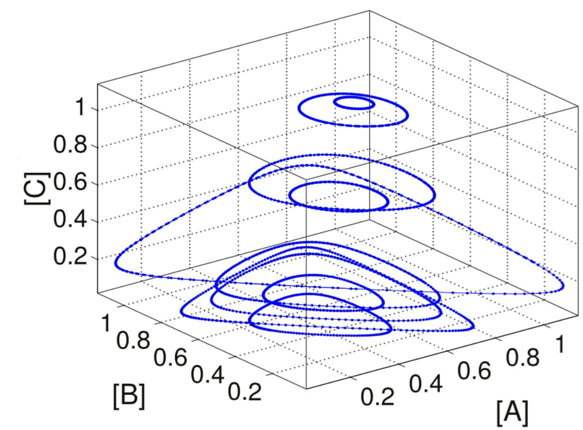
"Timing" analysis





$$Q \stackrel{\text{def}}{=} 2\pi \times \frac{\text{Extra Energy Applied}}{\text{Extra Energy Dissipated per Cycle}}$$





LPTV analysis

$$Q \Leftrightarrow \lambda_2 \text{ of } \mathbf{X}(T)$$

Does it take longer to injection lock a high-Q oscillator?